

MATEMATIK TAMBAHAN

MODUL SOALAN GALUS

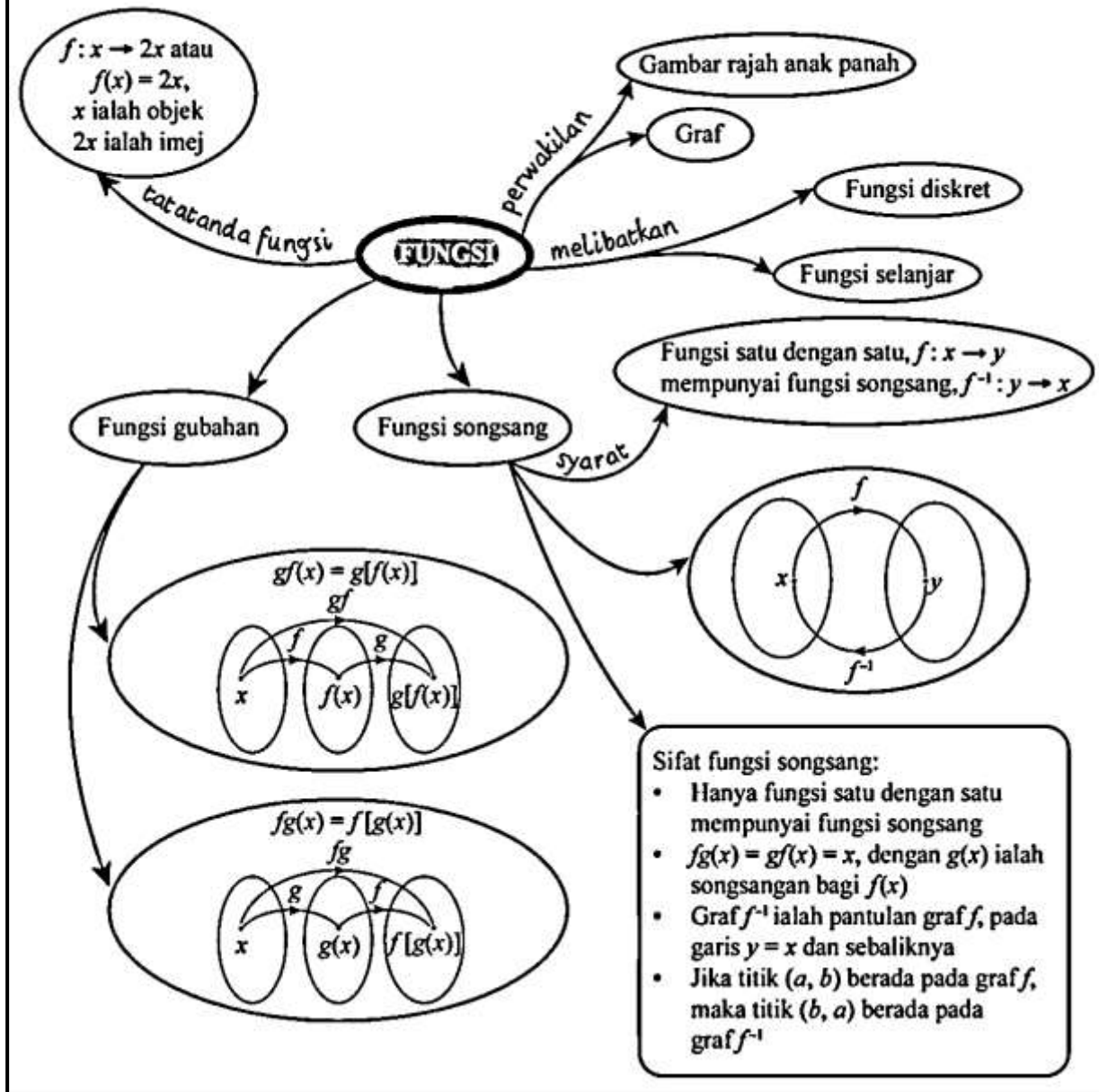
TINGKATAN 4

BIL	KANDUNGAN	MUKA SURAT
1	Bab 1 : Fungsi / <i>Functions</i>	3 – 10
2	Bab 2 : Fungsi Kuadratik / <i>Quadratic Functions</i>	11 – 26
3	Bab 3 : Sistem Persamaan / <i>Systems of Equations</i>	27 – 43
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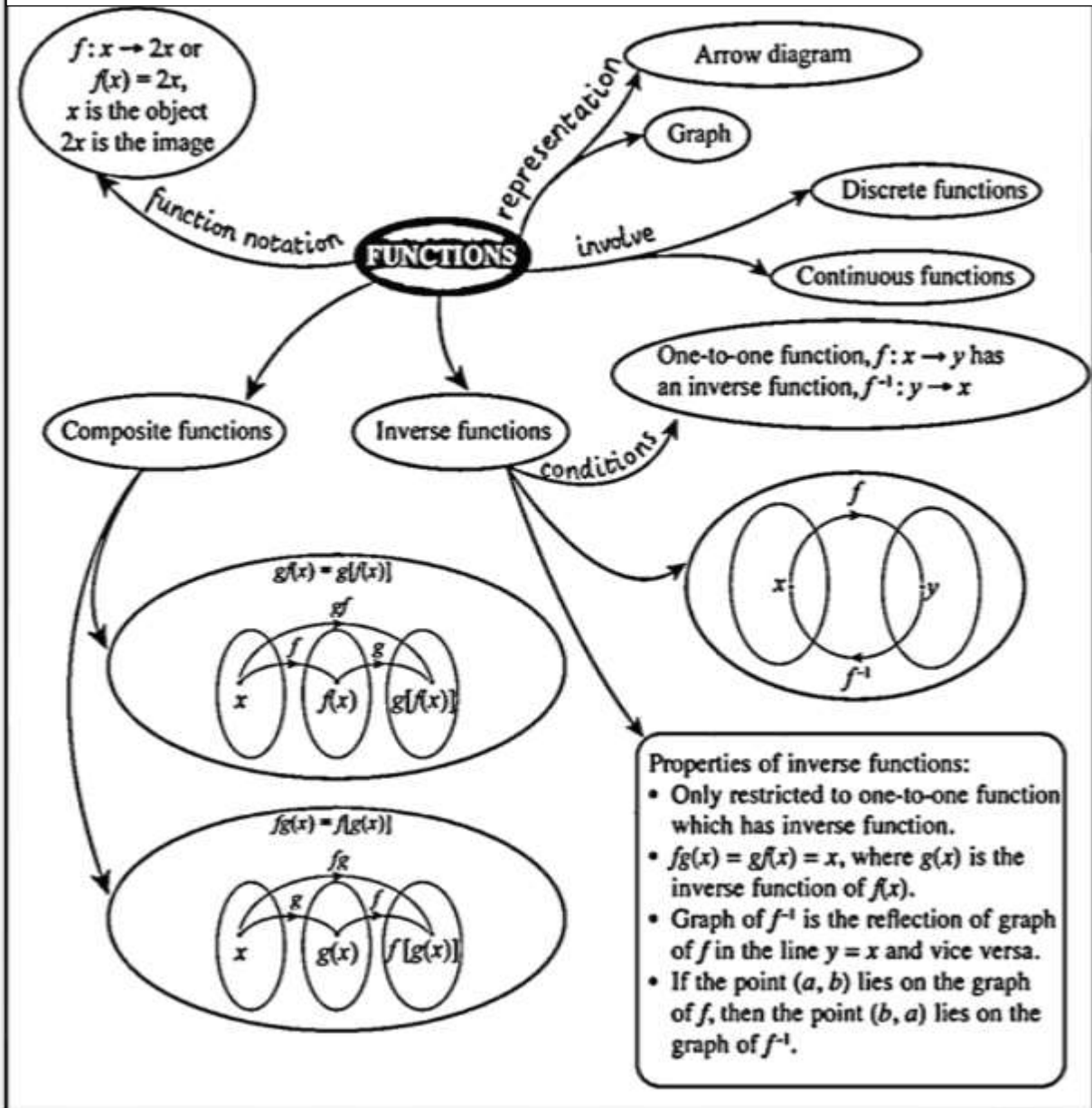
TINGKATAN 5

BIL	KANDUNGAN	MUKA SURAT
1	Bab 1 : Sukatan Membulat / <i>Circular Measure</i>	95 – 105
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FUNGSI



FUNCTIONS



TOPIK 1 (TINGKATAN4)

FUNGSI / FUNCTIONS

1. Diberi $f : x \rightarrow k - mx$. Cari $f^{-1}(x)$ dalam sebutan k dan m .
Given $f : x \rightarrow k - mx$. Find $f^{-1}(x)$ in term of k and of m .

[2 markah / 2 marks]

[Jawapan : $\frac{k-x}{m}$]

2. Diberi $f : x \rightarrow 5x + 6$ dan $g : x \rightarrow 2x - 1$, cari $gf(x)$.
Given $f : x \rightarrow 5x + 6$ and $g : x \rightarrow 2x - 1$, find $gf(x)$.

[2 markah / 2 marks]

[Jawapan : $10x + 11$]

3. Diberi $h : x \rightarrow 3x + 1$, cari $h^{-1}(5)$.
Given $h : x \rightarrow 3x + 1$, find $h^{-1}(5)$.

[2 markah / 2 marks]

[Jawapan : 4/3]

4. Diberi $g : x \rightarrow 3x - 1$, cari
Given $g : x \rightarrow 3x - 1$, find
- (a) $g(2)$
- (b) nilai p apabila $g^{-1}(p) = 11$.
the value of p when $g^{-1}(p) = 11$.

[3 markah / 3 marks]

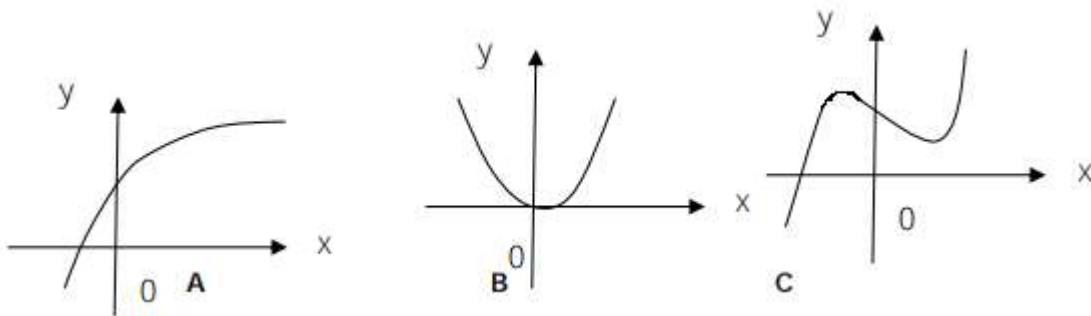
[Jawapan: (a) 5, (b) 32]

5. Diberi fungsi $f: x \rightarrow |3 - x|$, cari nilai-nilai x dengan keadaan $f(x) = 5$.
Given the function $f: x \rightarrow |3 - x|$, find the values of x such that $f(x) = 5$.

[2 markah / 2 marks]

[Jawapan : 8, -2]

6.



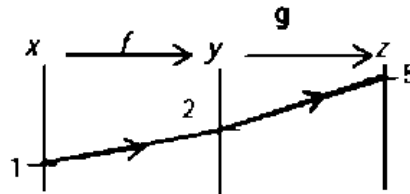
Graf yang manakah bukan merupakan hubungan satu kepada satu? Berikan alasan anda.

Which of the graph is not one - to - one relation? Give your reason.

[2 markah / 2 marks]

[Jawapan : B dan C kerana hubungan banyak kepada satu]

7. Rajah menunjukkan fungsi f memetakan x kepada y dan fungsi g memetakan y kepada z .
Diagram shows the function f mapped x to y and function g mapped y to z .



Nyatakan

State

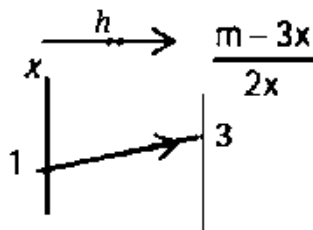
- (a) $f^{-1}(2)$
 (b) $gf(1)$

[2 markah / 2 marks]

[Jawapan : (a) 1 (b) 5]

8. Rajah menunjukkan fungsi $h : x \rightarrow \frac{m-3x}{2x}, x \neq 0$.

Diagram shows the function $h : x \rightarrow \frac{m-3x}{2x}, x \neq 0$.



Hitung nilai m .

Calculate the value of m .

[2 markah / 2 marks]

[Jawapan: 9]

9. Diberi $g : x \rightarrow \frac{3x + 10}{x}, x \neq 0$, cari

Given $g : x \rightarrow \frac{3x + 10}{x}, x \neq 0$, find

- (a) nilai - nilai x apabila $g(x)$ memetakan kepada diri sendiri.
the values of x when $g(x)$ mapped onto itself

[2 markah / 2 marks]

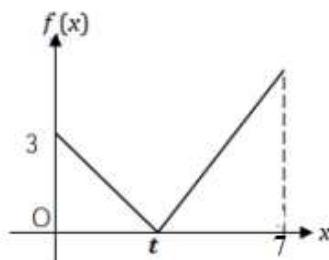
- (b) nilai k dengan keadaan $g(3 - k) = 4$.
the value of k when $g(3 - k) = 4$.

[2 markah / 2 marks]

[Jawapan : (a) 5, -2 (b) -7]

10. Rajah menunjukkan fungsi $f(x) = |3 - x|$ bagi domain $0 \leq x \leq 7$.

Diagram shows the function $f(x) = |3 - x|$ for the domain $0 \leq x \leq 7$.



Cari / Find

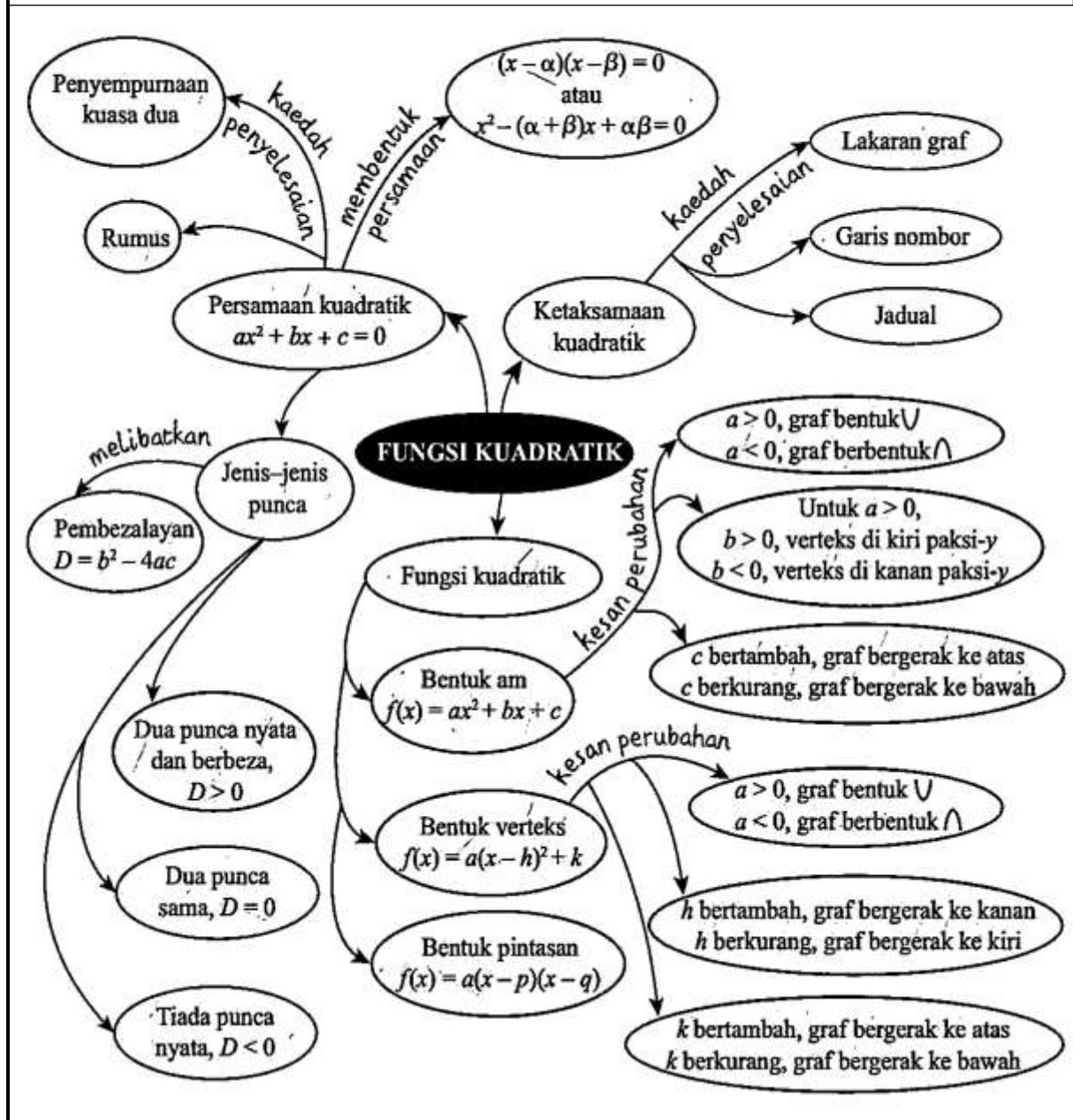
- (a) nilai t
the value of t

- (b) julat berdasarkan domain yang diberi
the range based on the domain given.

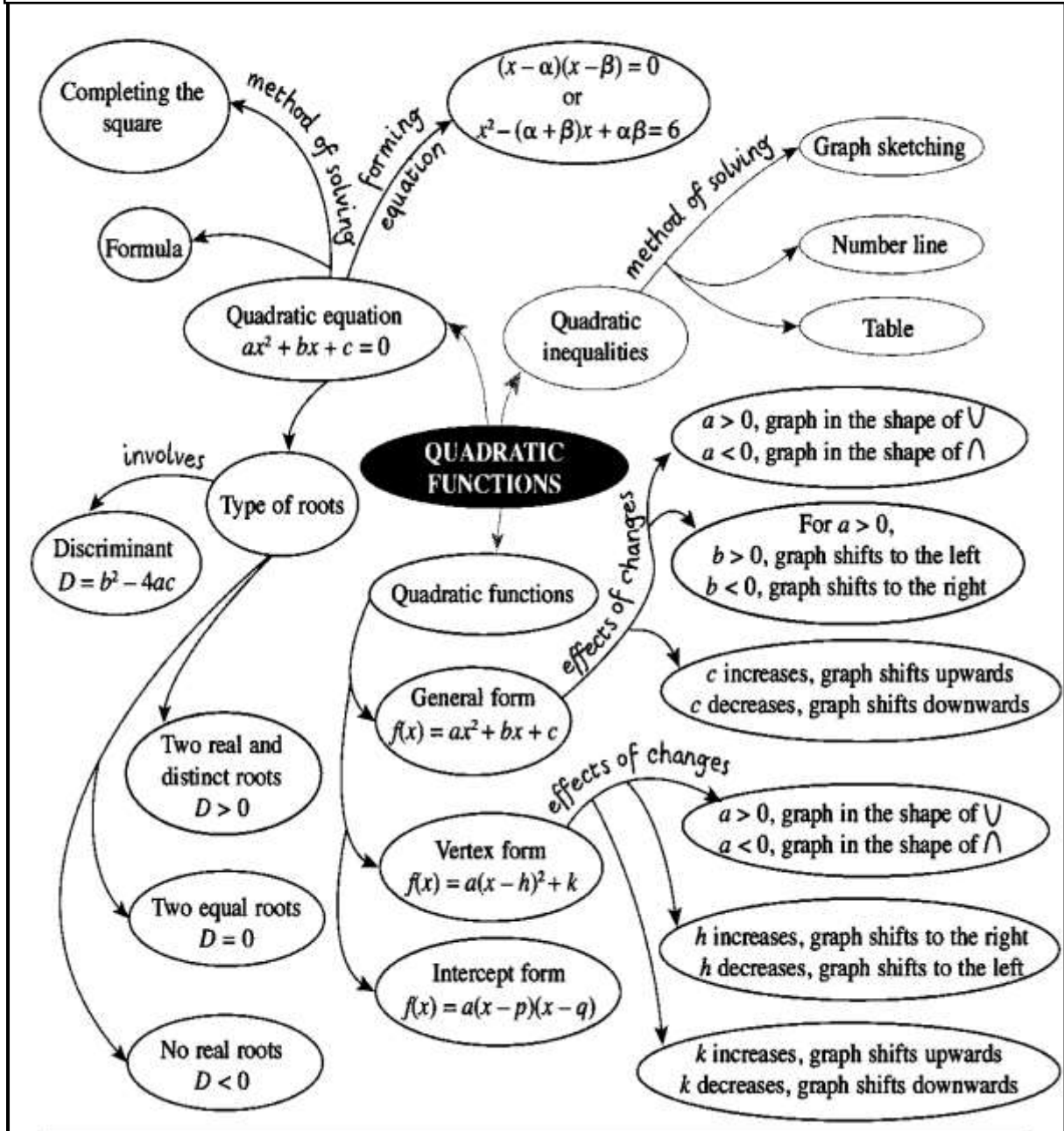
[4 markah / 4 marks]

[Jawapan : (a) 3 (b) $0 \leq f(x) \leq 4$]

FUNGSI KUADRATIK



QUADRATIC FUNCTIONS



TOPIK 2 (TINGKATAN 4)

FUNGSI KUADRATIK

QUADRATIC FUNCTIONS

1. (a) Selesaikan persamaan kuadrat $x(2x + 5) = 2x + 1$. Berikan jawapan betul kepada tiga tempat perpuluhan

Solve the quadratic equation $x(2x + 5) = 2x + 1$. Give your answer correct to three decimal places.

- (b) Diberi bahawa $f(x) = x^2 + 4x - 3$, cari nilai p dan nilai q bagi

$$f(x) = (x + p)^2 - q$$

Given that $f(x) = x^2 + 4x - 3$, find the value of p and value of q for which

$$f(x) = (x + p)^2 - q$$

Jawapan/ Answer

(a)

(b)

2. (a) Diberi -2 ialah satu daripada punca bagi persamaan kuadratik $2x^2 + 5x + p = 0$. Cari nilai p

Given -2 is one of the roots of the quadratic equation $2x^2 + 5x + p = 0$. Find the value of p .

- (b) Garis lurus $y = 2x + 3$ menyilang lengkung $y = x^2 - 4x + k$ pada dua titik berlainan. Cari julat nilai k

The straight line $y = 2x + 3$ intersects the curve $y = x^2 - 4x + k$ at two different points. Find the range of value of k

Jawapan/ Answer

(a)

(b)

3. (a) Jika persamaan kuadratik $2x^2 + 3h = 1 - 4kx$ tidak mempunyai punca.,
ungkapkan julat h dalam sebutan k .

*If the quadratic equation $2x^2 + 3h = 1 - 4kx$ has no roots, express the
range of h in terms of k .*

- (b) Diberi bahawa $(2, -4)$ adalah titik minimum bagi fungsi kuadratik

$$f(x) = (x + 2p)^2 + q - 1, \text{ cari nilai } p \text{ dan nilai } q.$$

Given that $(2, -4)$ is the minimum point of the quadratic function

$$f(x) = (x + 2p)^2 + q - 1, \text{ , find the value of } p \text{ and of } q.$$

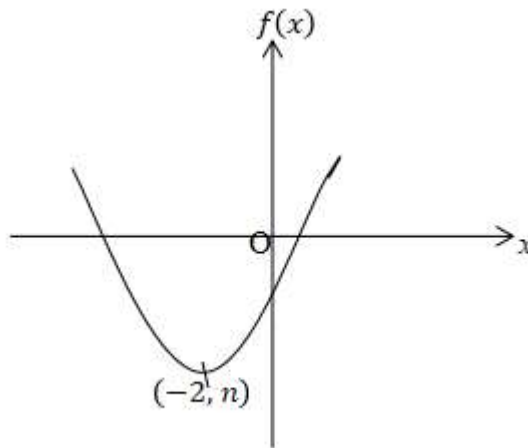
Jawapan/ Answer

(a)

(b)

4. (a) Rajah 4 menunjukkan graf bagi fungsi kuadratik $f(x) = 2(x + m)^2 - 3$, dengan keadaan m ialah pemalar. Lengkung $y = f(x)$ mempunyai titik minimum $(-2, n)$, dengan keadaan n ialah pemalar.

Diagram 4 shows the graph of a quadratic function $f(x) = 2(x + m)^2 - 3$, where m is a constant. The curve $y = f(x)$ has the minimum point $(-2, n)$, where n is a constant.



Rajah 4 / Diagram 4

Nyatakan / State

- (i) nilai m
the value of m
- (ii) nilai n
the value of n
- (iii) persamaan paksi simetri
the equation of the axis of symmetry

Jawapan/ Answer

- (a)

(b) Cari julat nilai x bagi $3x^2 - \frac{29}{4}x > 2$

Find the range of values of x for which $3x^2 - \frac{29}{4}x > 2$

Jawapan/ Answer

(b)

5. Diberi m dan n ialah punca-punca bagi persamaan kuadratik $2x^2 + 3x - 7 = 0$

Given m and n are the roots of the quadratic equation $2x^2 + 3x - 7 = 0$

a) Cari nilai bagi $m + n$ dan mn

Find the values of $m + n$ and mn

b) Seterusnya, bentukkan persamaan kuadratik dengan punca-punca $2m$ dan $2n$

Hence, form the quadratic equation which has the roots $2m$ and $2n$

Jawapan/ Answer

(a)

(b)

6. Ungkapkan fungsi kuadrat $f(x) = x^2 + 6x - 3$ dalam bentuk $f(x) = a(x + p)^2 + q$. Tentukan ,
Express the quadratic function $f(x) = x^2 + 6x - 3$ in the form of $f(x) = a(x + p)^2 + q$. State
- a) nilai-nilai a , p dan q
the values of a , p and q
- b) nilai maksimum atau minimum $f(x)$
the maximum or minimum value of $f(x)$
- c) sama ada graf $f(x)$ menyilang paksi- x
whether the graph of $f(x)$ intersects the x -axis

Jawapan/ Answer

(a)

(b)

(c)

7. Fungsi kuadratik $f(x) = 2x^2 - 3x + 1$ boleh diungkapkan dalam bentuk

$$f(x) = 2\left(x - \frac{3}{4}\right)^2 + k \text{ dengan keadaan } k \text{ ialah pemalar.}$$

The quadratic function $f(x) = 2x^2 - 3x + 1$ can be expressed in the form of

$$f(x) = 2\left(x - \frac{3}{4}\right)^2 + k, \text{ where } k \text{ is a constant.}$$

a) Cari nilai k

Find the value of k

b) Lakar graf fungsi $f(x)$

Sketch the graph of the function $f(x)$

Jawapan/ Answer

a)

b)

8. (a) Diberi bahawa 2α dan 2β adalah punca-punca persamaan kuadratik $3x^2 = 6 - 9x$. Bentuk persamaan kuadratik dengan punca-punca, α dan β

It is given that 2α and 2β are the roots of the quadratic equation

$3x^2 = 6 - 9x$. Form a quadratic equation with roots, α and β

- (b) Diberi beza antara punca-punca bagi persamaan kuadratik $2x^2 + 8x = p - 3$ ialah 2, dengan keadaan p ialah pemalar. Cari nilai p .

Given the difference between the roots of the quadratic equation

$2x^2 + 8x = p - 3$ is 2, where p is a constant. Find the value of p .

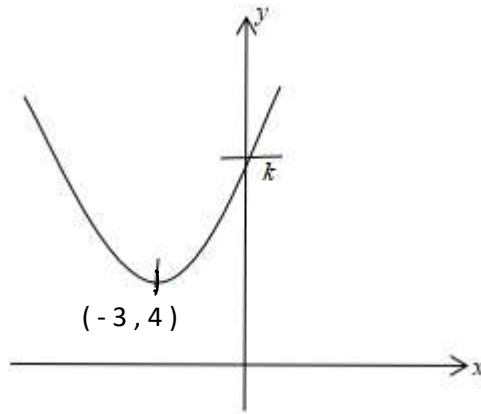
Jawapan/ Answer

a)

b)

9. (a) Rajah 9 menunjukkan graf fungsi kuadratik $f(x) = (x + p)^2 + 4$

Diagram 9 shows the graph of quadratic function $f(x) = (x + p)^2 + 4$



Rajah 9/ Diagram 9

Cari nilai k dan nilai p

Find the values of k and of p

- (b) Diberi fungsi kuadratik $f(x) = px(x - 1) - px + 2$. Cari julat nilai p apabila $f(x)$ adalah positif

Given a quadratic function $f(x) = px(x - 1) - px + 2$. Find the range of p when $f(x)$ is positive

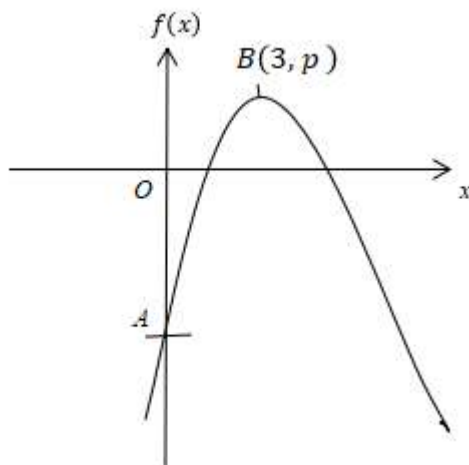
Jawapan/ Answer

a)

b)

10. Rajah 10 menunjukkan lengkung bagi fungsi kuadratik $f(x) = -x^2 + kx - 8$. Lengkung itu mempunyai titik maksimum pada $B(3, p)$ dan memotong paksi $f(x)$ pada titik A.

Diagram 10 shows the curve of a quadratic function $f(x) = -x^2 + kx - 8$. The curve has a maximum point at $B(3, p)$ and intersects the $f(x)$ -axis at point A.



Rajah 10 / Diagram 10

- Nyatakan koordinat A
State the coordinates of A
- Dengan menggunakan kaedah penyempurnaan kuasa dua, cari nilai k dan nilai p
By using the method of completing the square, find the value of k and p
- Tentukan julat nilai x , jika $f(x) \leq -8$
Determine the range of values of x , if $f(x) \leq -8$

Jawapan/ Answer

11. Diberi bahawa fungsi kuadratik $f(x) = 4x^2 - 16x - 9$

It is given that the quadratic function $f(x) = 4x^2 - 16x - 9$

a) Menggunakan kaedah penyempurnaan kuasa dua, ungkapkan $f(x)$ dalam bentuk

$$f(x) = a(x + p)^2 + q$$

Using completing the square method, express $f(x)$ in the form of

$$f(x) = a(x + p)^2 + q$$

b) Cari nilai maksimum atau minimum bagi fungsi $f(x)$

Find the maximum or minimum value of $f(x)$

c) Lakar graf bagi $f(x) = 4x^2 - 16x - 9$

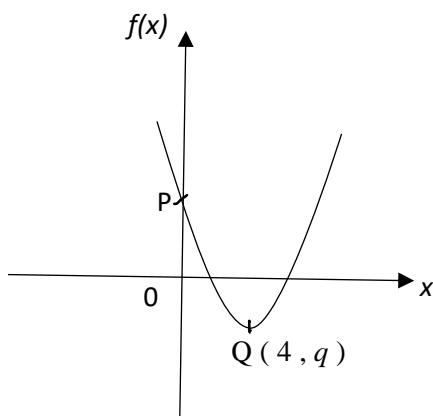
Sketch the graph of $f(x) = 4x^2 - 16x - 9$

d) Nyatakan persamaan lengkung apabila graf tersebut dipantulkan pada paksi-x

State the equation of the curve when the graph is reflected in the x-axis

Jawapan/ Answer

12. Rajah 12 menunjukkan lengkung bagi fungsi $f(x) = x^2 + kx + 12$. Lengkung itu mempunyai titik minimum pada $Q(4, q)$ dan bersilang dengan paksi- $f(x)$ pada titik P
- Diagram 12 shows the curve of a quadratic function $f(x) = x^2 + kx + 12$. The curve has a minimum point at $Q(4, q)$ and intersect the $f(x)$ -axis at point P*



Rajah 12 / Diagram 12

- Nyatakan koordinat P
State the coordinates of P
- Dengan penyempurnaan kuasa dua, cari nilai q dan nilai k
By using completing the square, find the values of q and k
- Tentukan julat nilai-nilai x , jika diberi $f(x) \leq 5$
Determine the range of values of x given that $f(x) \leq 5$

Jawapan/ Answer

SKEMA FUNGSI KUADRATIK

QUADRATIC FUNCTION MARKING SCHEME

NO	SKEMA	MARKS
1(a)	$x = 0.281, x = -1.781$	3
(b)	$p = 2, q = 7$	3
2(a)	$p = 2$	2
(b)	$k < 12$	3
3(a)	$h > \frac{2k^2 + 1}{3}$	3
(b)	$p = -1, q = -3$	3
4(a)	$m = 2, n = -3, x = -2$	3
(b)	$x < -\frac{1}{4}, x > \frac{8}{3}$	3
5(a)	$m + n = -\frac{3}{2}, mn = -\frac{7}{2}$	3
(b)	$x^2 + 3x - 14 = 0$	3
6(a)	$a = 1, p = 3, q = -12$	3
(b)	<i>Minimum</i> = -12	1
(c)	Ya/Yes	2
7(a)	$k = -\frac{1}{8}$	3
(b)		4

8(a) $2x^2 + 3x - 1 = 0$ 3

(b) $p = -3$ 4

9(a) $k = 13, p = 3$ 3

(b) $0 < p < 2$ 3

10(a) $A(0, -8)$ 1

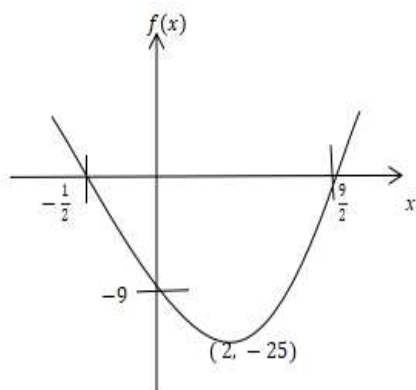
(b) $k = 6, p = 1$ 4

(c) $x \leq 0, x \geq 6$ 3

11(a) $4(x - 2)^2 - 25$ 2

(b) *Minimum value* = -25 1

(c)



3

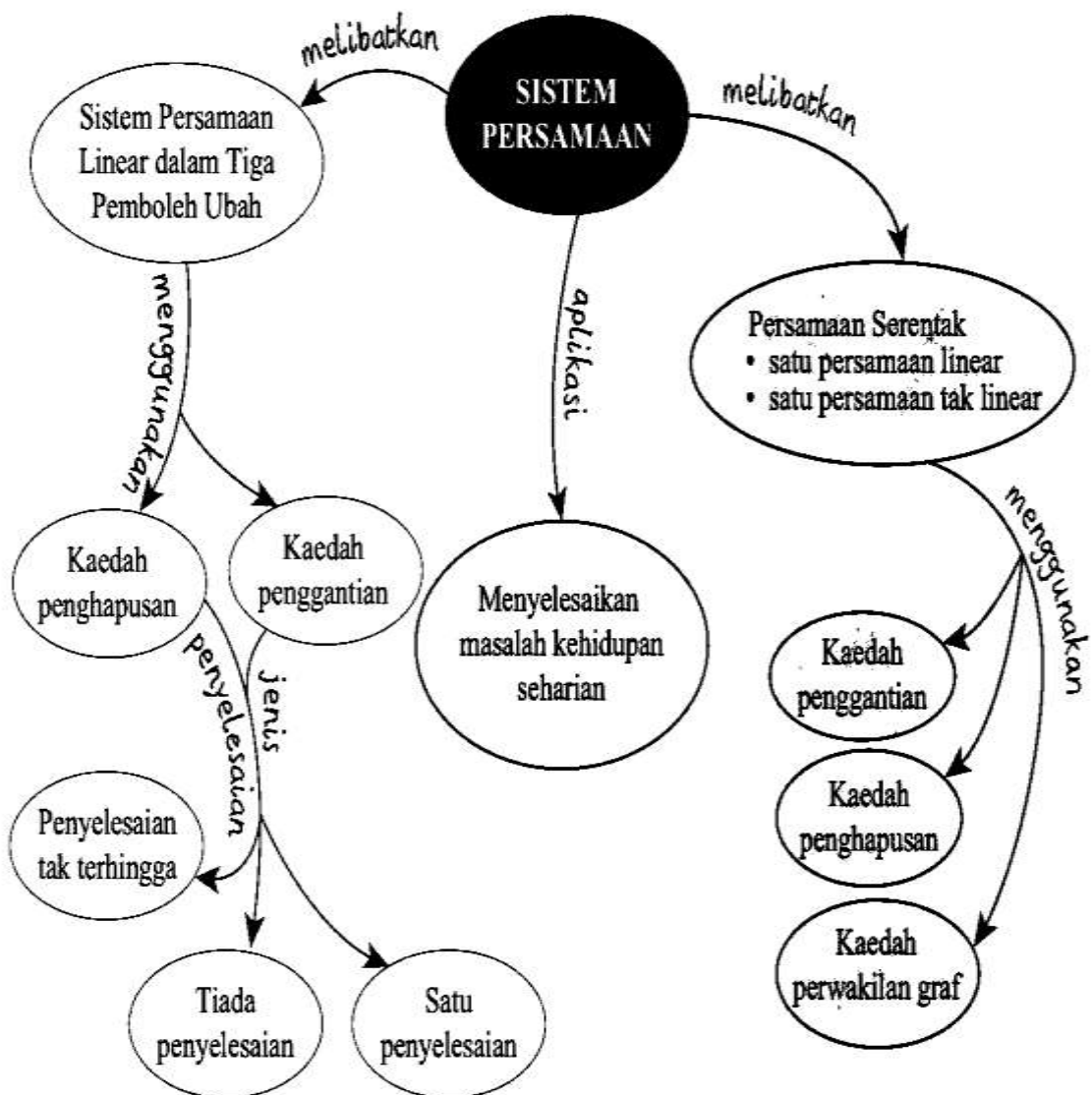
(d) $f(x) = -4x^2 + 16x + 9$ 1

12(a) $P(0, 12)$ 1

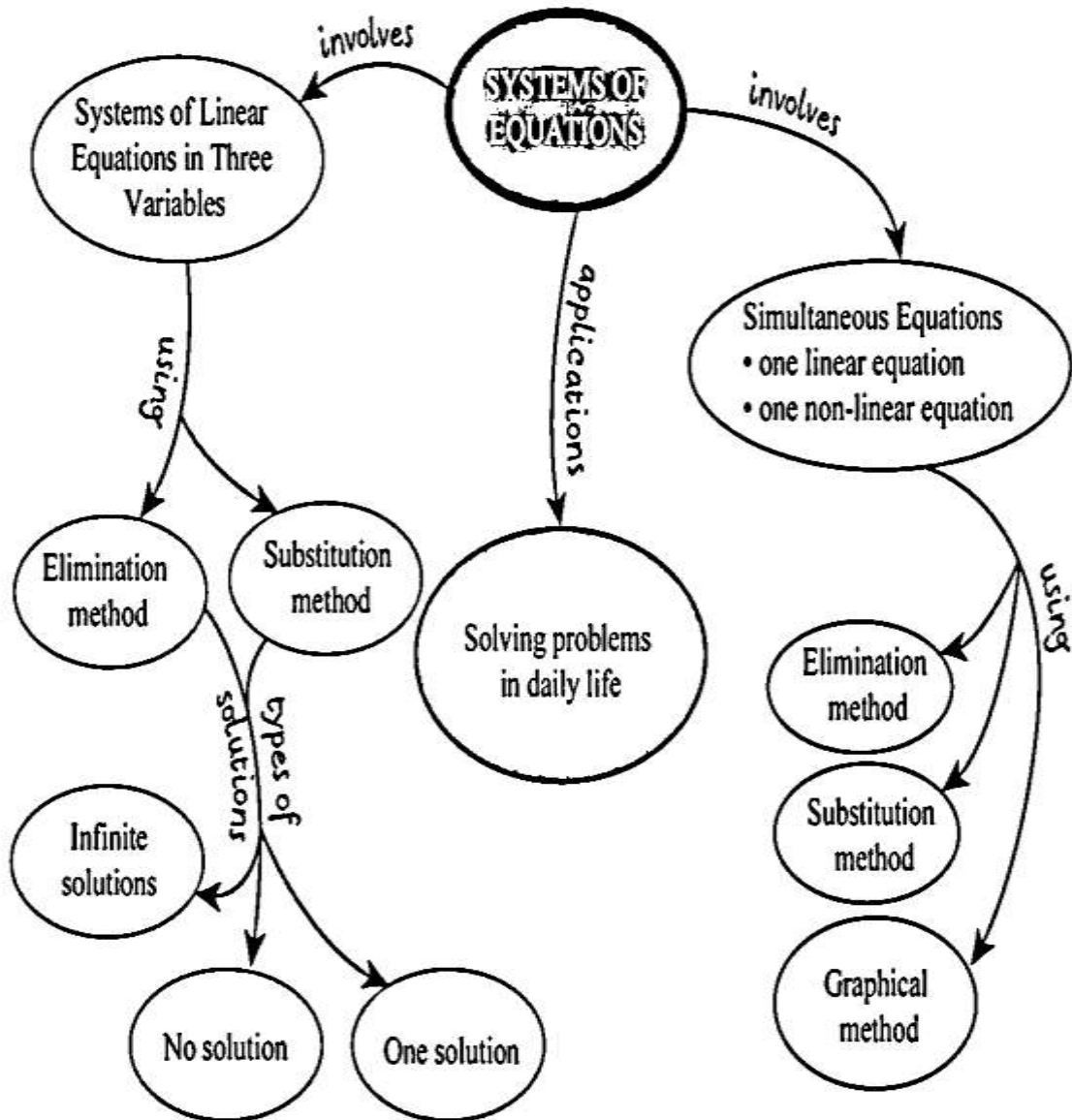
(b) $k = -8, q = -4$ 4

(c) $1 \leq x \leq 7$ 3

SISTEM PERSAMAAN



SYSTEMS OF EQUATIONS



TOPIK 3 (TINGKATAN 4)

SISTEM PERSAMAAN / SYSTEMS OF EQUATION

3.1: Sistem Persamaan Linear dalam Tiga Pemboleh Ubah

Linear Equation Systems in Three Variables

Contoh 1:

Example 1:

Selesaikan sistem persamaan linear yang berikut;

Solve the following system of linear equations.

$x + y - z = 4$ persamaan (1) / equation (1)

$x - 2y + 3z = -6$ persamaan (2)/ equation (2)

$2x + 3y + z = 7$ persamaan (3) / equation (3)

Kaedah 1: Kaedah Penghapusan

Kaedah 2:Kaedah Penggantian

Method 1: Elimination Method

Method 2: Substitution Method

1 Pilih mana-mana 2 persamaan
Choose any 2 equations,

$x + y - z = 4$ persamaan / equation (1)

$x - 2y + 3z = -6$..persamaan / equation (2)

$x + y - z = 4$ persamaan/equation (1)

$2x + 3y + z = 7$ persamaan / equation (3)

$x - 2y + 3z = -6$...persamaan/equation (2)

1

2 Hapuskan pemboleh ubah x
Eliminate variable x using subtraction

Pilih mana-mana 1 persamaan yang mempunyai pemalarnya satu untuk mana-mana pemboleh ubah

Choose any 1 equation that have coefficient is one for any variables

Persamaan/equation (1)- Persamaan/ equation (2):

$$\begin{array}{r} x + y - z = 4 \\ x - 2y + 3z = -6 \\ \dots\dots\dots \end{array}$$

3y - 4z = 10...persamaan (4)/ equation 4

3

Ulangi langkah 1 dengan menggunakan 2 persamaan yang lain

Repeat step 1 using any other 2 equations

$x - 2y + 3z = -6$..persamaan /equation (2)

$2x + 3y + z = 7$ persamaan/ equation (3)

4

Samakan mana-mana pekali dalam kedua-dua persamaan tersebut (pada pekali x) dengan operasi darab

Equalize any quotation in both equations (examples quation for x) using multiplication

$2(x - 2y + 3z = -6)$...persamaan / equation(2)

$2x + 3y + z = 7$...persamaan / equation (3)

5

Hapuskan pemboleh ubah x
Eliminate variable x using subtraction

Daripada persamaan (1)/from equation (1)

$x = -y + z + 4$ persamaan/equation (4)

Peringatan!
Persamaan (4) tidak boleh digantikan dalam persamaan (1) kerana persamaan (4) adalah daripada persamaan (1)

2

Gantikan persamaan (4) ke dalam persamaan (2):

Substitute equation (4) into equation (2)

$(-y + z + 4) - 2y + 3z = -6$

$-3y + 4z = -10$...Persamaan/ equation (5)

3

Gantikan persamaan (4) ke dalam persamaan (3):

Substitute equation (4) into equation (3)

$2(-y + z + 4) + 3y + z = 7$

$-2y + 2z + 8 + 3y + z = 7$

$y + 3z = -1$ Persamaan / equation (6)

Pilih mana-mana satu persamaan daripada persamaan (5) dan persamaan (6)

$2x - 4y + 6z = -12$)...daripada persamaan (2) *Choose any equations from equation (5) or equation (6)*

- Daripada persamaan (6),

$2x + 3y + z = 7$ Persamaan/ equation (3) *From equation (6)*

.....
 $y = -3z - 1$ Persamaan/ equation (7)
 $-7y + 5z = -19$...persamaan/ equation (5)

Kembangkan/expand

$7(3y - 4z = 10)$7 darab persamaan (4)/ 7 multiply by equation (4)

$-3(-7y + 5z = -19)$ -3 darab persamaan (5)/-3 multiply by equation (5)

Hapuskan y ,

Eliminate variable of y

$$21y - 28z = 70$$

$$21y - 15z = 57$$

.....

$$-13z = 13$$

$$z = \frac{13}{-13}$$

$$z = -1$$

Gantikan $z = -1$ dalam $3y - 4z = 10$

Substitute $z = -1$ into $3y - 4z = 10$

$$3y - 4(-1) = 10$$

$$3y = 10 - 4$$

$$3y = 6$$

$$y = 2$$

Ingat!

Pilih persamaan yang mempunyai salah satu pemboleh ubah dalam persamaan tersebut mempunyai pekali 1 (jika tiada pilihan, boleh guna mana-mana persamaan)

4

Gantikan persamaan (7) ke dalam persamaan (5):

Substitute equation (7) into equation (5)

$$-3(-3z - 1) + 4z = -10$$

$$9z + 3 + 4z = -10$$

$$13z = -13$$

$$z = -1$$

Peringatan!

Persamaan (7) tidak boleh digantikan dalam persamaan (6) kerana persamaan (7) adalah daripada persamaan (6)

Gantikan $z = -1$ dan $y = 2$ dalam persamaan $x + y - z = 4$

Substitute $z = -1$ and $y = 2$ into
 $x + y - z = 4$

$$x + 2 - (-1) = 4$$

$$x + 3 = 4$$

$$x = 1$$

Jawapan/Answer:

$$x = 1, y = 2 \text{ dan } /and \ z = -1$$

5

Gantikan $z = -1$ dalam persamaan (7)

Substitute $z = -1$ into equation (7)

$$y = -3(-1) - 1$$

$$y = 2$$

6

Gantikan $z = -1$ dan $y = 2$ dalam persamaan (4):

Substitute $z = -1$ and $y = 2$ into equation (4)

$$x = -(2) + (-1) + 4$$

$$x = 1$$

Jawapan/Answer:

$$x = 1, y = 2 \text{ dan } z = -1$$

Latihan

Exercise

(1) Selesaikan sistem persamaan linear yang berikut.

Solve the following systems of linear equations

$$4p - 2q + 3r = 1$$

$$p + 3q - 4r = -7$$

$$3p + q + 2r = 5$$

(2) Selesaikan sistem persamaan linear yang berikut.

Solve the following systems of linear equations

$$x + y - 2z = 1$$

$$x + 2y + z = 3$$

$$2x + y - 3z = 4$$

[Jawapan : $p = -1$, $q = 2$, $z = 3$]

[Jawapan : $x = 4$, $y = -1$, $z = 1$]

(3) Selesaikan sistem persamaan linear yang berikut.

Solve the following systems of linear equations

$$2x - 4y - 5z = -6$$

$$3x + 2y - 10z = -4$$

$$x + 8y - 5z = 3$$

(4) Selesaikan sistem persamaan linear yang berikut.

Solve the following systems of linear equations

$$7a + 5b - 3c = 16$$

$$3a - 5b + 2c = -8$$

$$5a + 3b - 7c = 0$$

$$\left[\text{Jawapan : } x = -3, \quad y = \frac{1}{2}, \quad z = -\frac{2}{5} \right]$$

$$[\text{Jawapan : } a = 1, \quad b = 3, \quad c = 2]$$

(5) Selesaikan sistem persamaan linear yang berikut.

Solve the following systems of linear equations

$$x + y + z = 3$$

$$x + z = 2$$

$$2x + y + z = 5$$

(6) Selesaikan sistem persamaan linear yang berikut.

Solve the following systems of linear equations

$$2x - y - 2z = 5$$

$$x + y + 4z = 0$$

$$3x - 2y - 3z = 5$$

[Jawapan : $x = 2$, $y = 1$, $z = 0$] [Jawapan : $x = 3$, $y = 5$, $z = -2$]

(7) Selesaikan sistem persamaan linear yang berikut.

Solve the following systems of linear equations

$$\begin{aligned}x + y + z &= 2 \\6x - 4y + 5z &= 31 \\5x + 2y + 2z &= 13\end{aligned}$$

(8) Selesaikan sistem persamaan linear yang berikut.

Solve the following systems of linear equations

$$\begin{aligned}x - y + 2z &= 3 \\x + y - 3z &= -10 \\2x + y - z &= -6\end{aligned}$$

[Jawapan : $x = 3$, $y = -2$, $z = 1$]

[Jawapan : $x = -2$, $y = 1$, $z = 3$]

(9) Selesaikan sistem persamaan linear yang berikut.

Solve the following systems of linear equations

$$x + 2y + 5z = -17$$

$$2x - 3y + 2z = -16$$

$$3x + y - z = 3$$

(10) Selesaikan sistem persamaan linear yang berikut.

Solve the following systems of linear equations

$$3x - y - z = -120$$

$$y - 2z = 30$$

$$x + y + z = 180$$

[Jawapan : $x = -1$, $y = 2$, $z = -4$] [Jawapan : $x = 15$, $y = 120$, $z = 45$]

3.2 : Persamaan serentak melibatkan satu persamaan linear dan satu persamaan tidak linear.

Simultaneous equation with one linear equation and one non-linear equation

Contoh 1:

$$P^2 + 2p - 4 - 8 = 0$$

Example 1

$$P^2 + 2p - 12 = 0$$

Selesaikan persamaan serentak

$$a = 1, b = 2, c = -12$$

$$p - m = 2 \quad \text{dan} \quad P^2 + 2m = 8$$

Guna formula ini,

Use this formulae

Berikan jawapan betul kepada tiga tempat perpuluhan.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Solve the simultaneous equations

$$p - m = 2 \quad \text{dan} \quad P^2 + 2m = 8$$

$$p = \frac{-2 \pm \sqrt{2^2 - 4(1)(-12)}}{2(1)}$$

Give your answers correct to three decimal places.

$$p = \frac{-2 \pm \sqrt{52}}{2}$$

Penyelesaian :

$$p = 2.606 \quad , \quad p = -4.606$$

$$p - m = 2 \dots \text{persamaan/equation (1)}$$

Gantikan $p = 2.606$ dalam persamaan (3)

$$P^2 + 2m = 8 \dots \text{persamaan/equation (2)}$$

Substitute $p = 2.606$ into equation (3)

Daripada persamaan (1),

$$m = 2.606 - 2$$

From equation (1)

$$m = 0.606$$

$$m = p - 2 \dots \dots \text{persamaan/equation (3)}$$

Gantikan $p = -4.606$ dalam persamaan (3)

Substitute $p = -4.606$ into equation (3)

$$m = -4.606 - 2$$

$$m = -6.606$$

Gantikan persamaan (3) dalam persamaan (2):

Substitute equation (3) into equation (2)

$$p = 2.606 \quad , \quad m = 0.606$$

$$P^2 + 2(p - 2) = 8$$

$$p = -4.606, \quad m = -6.606$$

Latihan

Exercise

- (1) Selesaikan sistem persamaan linear yang berikut.

Solve the following systems of linear equations

$$x + 2y = 2 \quad \text{dan} \quad 2y^2 - xy = 6$$

$$\left[\text{Ans: } x = -1, y = \frac{3}{2}, \quad x = 4, y = -1 \right]$$

- (2) Selesaikan sistem persamaan linear yang berikut.

Solve the following systems of linear equations

$$1 - x = 3y \quad \text{dan} \quad x^2 + y^2 - xy = 21$$

$$\left[\text{Ans: } x = 4, y = -1, \quad x = \frac{-47}{13}, \quad y = \frac{20}{13} \right]$$

(3) Selesaikan sistem persamaan linear yang berikut.

Solve the following systems of linear equations

$$3x + 2y + 1 = 0 \quad \text{dan} \quad xy - y^2 + 2 = 0$$

$$\left[\text{Ans: } x = -1, y = 1, x = \frac{7}{15}, y = \frac{-6}{5} \right]$$

(4) Selesaikan persamaan serentak berikut dan berikan jawapan kepada dua tempat perpuluhan

Solve the simultaneous equations Give your answers correct to two decimal places.

$$\begin{aligned} 3y^2 &= 7 - \frac{x}{2} \\ x - 4y &= 9 \end{aligned}$$

$$\begin{aligned} (\text{Ans : } x = 11.56, y = 0.64 \text{ and } x = 3.76, \\ y = -1.31) \end{aligned}$$

(5) Selesaikan persamaan serentak berikut dalam 4 angka bererti.

Solve the simultaneous equations correct to four significant figures.

$$3x - 2y = 6 \text{ dan/ and } 2x^2 + 4xy - 2 = 0$$

(Ans : $x = 1.651, y = -0.5235 / -0.5229$ and $x = -0.1513, y = -3.227$)

(6) Selesaikan persamaan serentak berikut

Solve the simultaneous equations

$$\begin{aligned}x^2 + xy - 40 &= 0 \\x - 3y + 4 &= 0\end{aligned}$$

$\left[\text{Ans: } x = 5, y = 3, x = -6, y = \frac{-2}{3} \right]$

(7) Selesaikan persamaan serentak berikut betul kepada tiga tempat perpuluhan.

Solve the simultaneous equations correct to three decimal places.

$$x + 2y = 1$$

$$\frac{3}{x} - \frac{2}{y} = 5$$

(Ans : $x = 0.284$, $y = 0.358$ and $x = 2.116$,
 $y = -0.558$)

(8) Selesaikan persamaan serentak berikut betul kepada tiga tempat perpuluhan.

Solve the simultaneous equations correct to three decimal places.

$$y - 2x + 1 = 0$$

$$x^2 - 2y^2 - 3y + 2 = 0$$

Ans : $x = 0.813$, $y = 0.626$ and $x = -0.527$,
 $y = -2.054$)

- (9) Selesaikan persamaan serentak berikut dan berikan jawapan kepada dua tempat perpuluhan

Solve the simultaneous equations .Give your answers correct to two decimal places.

$$x - 2y = 7$$

$$xy - x = 9y$$

(Ans : $x = 13.24$, $y = 3.12$ and $x = 4.76$, $y = -1.12$)

- (10) Selesaikan persamaan serentak berikut betul kepada tiga tempat perpuluhan.

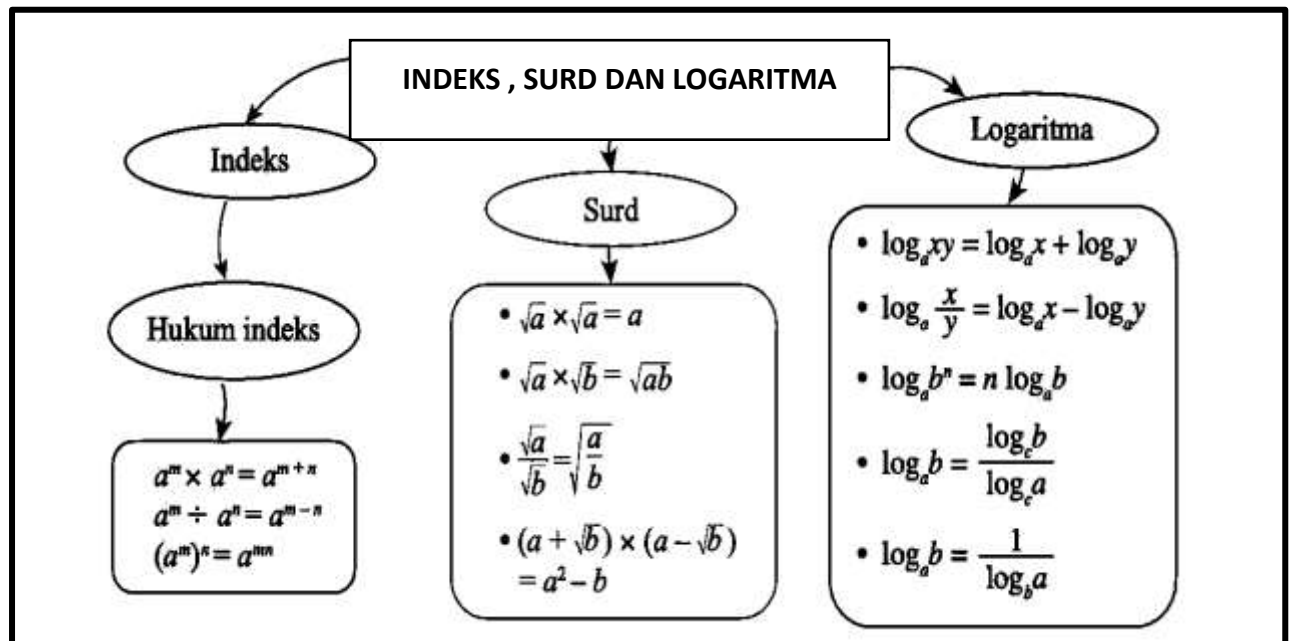
Solve the simultaneous equations correct to three decimal places.

$$k - 3p = -1$$

$$p + pk - 2k = 0$$

(Ans : $p = 1.577$, $k = 3.731$ and $p = 0.423$, $k = 0.269$)

INDEKS , SURD , DAN LOGARITMA



TOPIK 4 (TINGKATAN 4)
INDEKS, SURD, DAN LOGARITMA
INDICES, SURDS AND LOGARITHMS

- 1 Selesaikan persamaan:

Solve the equation:

$$3^{x+2} - 3^x = \frac{8}{27}$$

[3 marks /3 markah]

Jawapan : $x = -3$

- 2 Diberi bahawa $\log_2 x = a$ dan $\log_2 y = b$, ungkapkan $\log_2 x^2 y$ dalam sebutan a dan b .

Given that $\log_2 x = a$ and $\log_2 y = b$, express $\log_2 x^2 y$ in terms of a and b .

[2 markah / 2marks]

Jawapan : $2a + b$

3 Permudahkan:

Simplify:

$$\sqrt{3}(17 - 5\sqrt{3})$$

[2 markah / 2 marks]

Jawapan : $17\sqrt{3} - 15$

4 Selesaikan persamaan berikut.

Solve the following equations.

(a) $\sqrt{4^{x+4}} = \frac{1}{2^{x+3}}$

(b) $4^{2x-1} + 4^{2x} = 5$

[6 markah / 6 marks]

Jawapan : (a) $x = -\frac{7}{2}$ (b) $x = \frac{1}{2}$

- 5 (a) Tuliskan $\sqrt{3a} \times \sqrt{5a}$ sebagai surd tunggal.

Write $\sqrt{3a} \times \sqrt{5a}$ as a single surd.

- (b) Permudahkan penyebut bagi

Rationalize the denominator of

(i) $\frac{2}{\sqrt{x}}$

(ii) $\frac{x}{2+\sqrt{3}}$

[4 markah / 4 marks]

Jawapan : (a) $\sqrt{15a}$ (b)(i) $\frac{2\sqrt{x}}{x}$ (ii) $(2-\sqrt{3})x$

- 6 Diberi $\log_a 3 = x$ dan $\log_a 5 = y$, ungkapkan $\log_a \frac{45}{a^3}$ dalam sebutan x dan y .

Given $\log_a 3 = x$ and $\log_a 5 = y$, express $\log_a \frac{45}{a^3}$ in terms of x and y .

[3 markah / 3 marks]

Jawapan : $2x + y - 3$

7 Jika $3 - \log_{10} x = 2 \log_{10} y$, ungkapkan y dalam sebutan x .

If $3 - \log_{10} x = 2 \log_{10} y$, express y in terms of x .

[3 markah / 3 marks]

Jawapan : $y = \sqrt{\frac{1000}{x}}$

8 Selesaikan persamaan $\log_4 x = \log_2 3$.

Solve the equation $\log_4 x = \log_2 3$.

[3 markah / 3 marks]

Jawapan : $x = 9$

9 Diberi $2^{x+2} \cdot 3^{x-2} \cdot 5^2 = 6^{3x}$, cari nilai bagi 6^x .

Given $2^{x+2} \cdot 3^{x-2} \cdot 5^2 = 6^{3x}$, find the value of 6^x .

[3 markah / 3 marks]

Jawapan : $\frac{10}{3}$

10 Diberi $2\log_3 m^2 - 3\log_3 n - \log_9 p^8 + \log_3 q = 3$, ungkapkan q dalam sebutan m , n dan p .

Given $2\log_3 m^2 - 3\log_3 n - \log_9 p^8 + \log_3 q = 3$, express q in terms of m , n and p .

[3 markah / 3 marks]

Jawapan : $q = \frac{27n^3p^4}{m^4}$

11 (a) Diberi persamaan $2\log_y 3x = 1$. Nyatakan y dalam sebutan x .
Given the equation $2\log_y 3x = 1$. State y in terms of x .

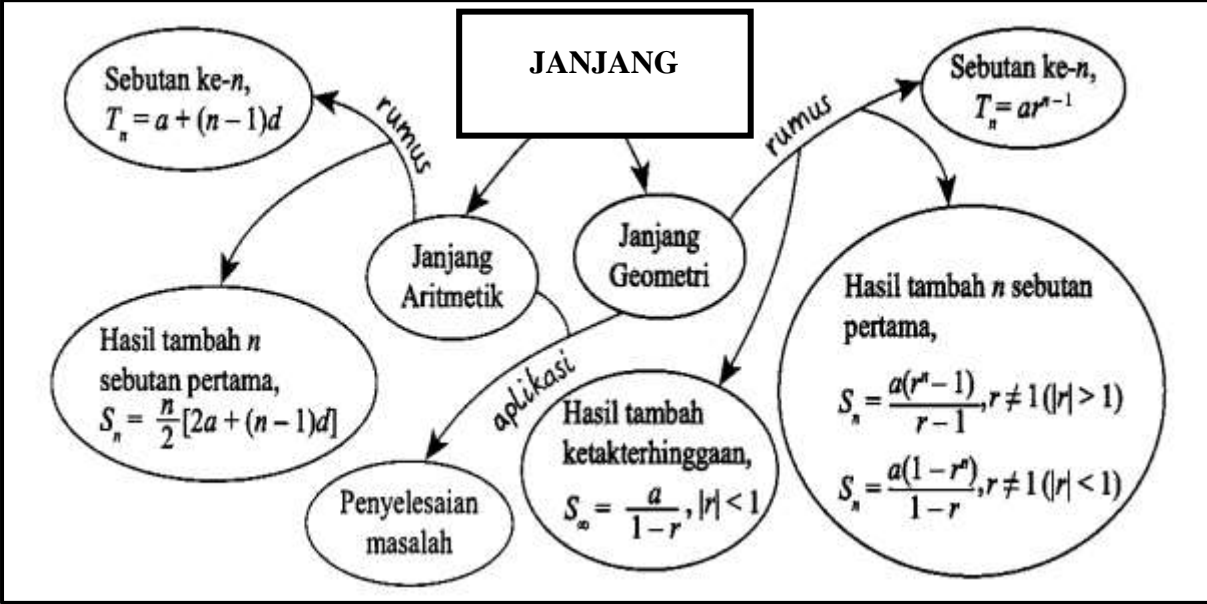
(b) Selesaikan persamaan $\log_3 x = \log_9(2x + 3)$.

Solve the equation $\log_3 x = \log_9(2x + 3)$.

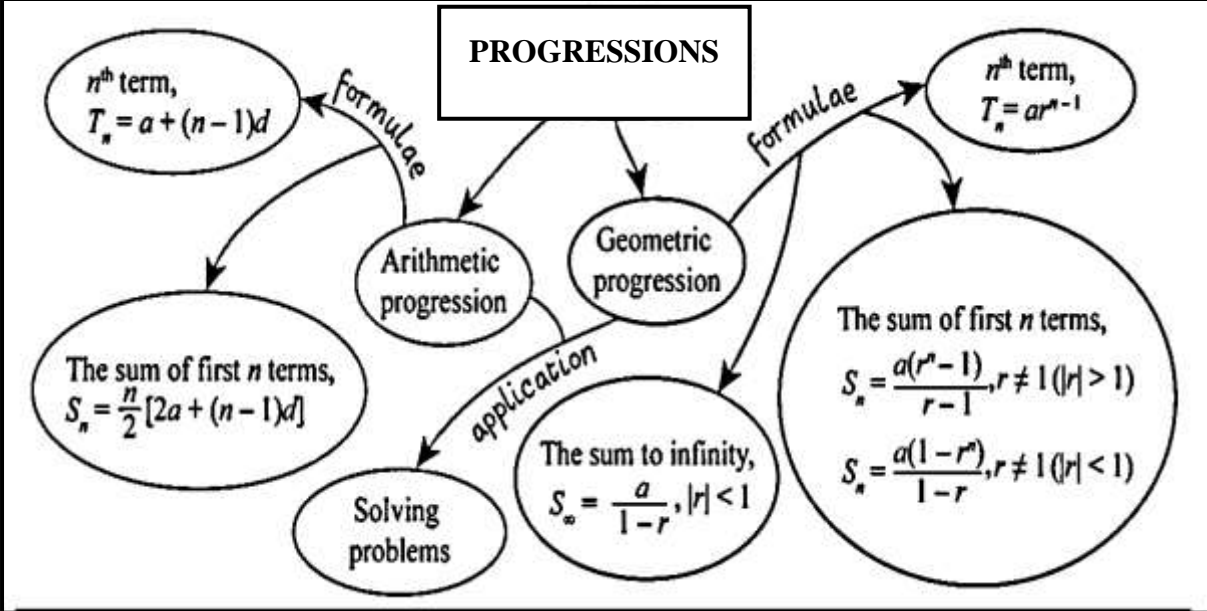
[4 markah / 4 marks]

Jawapan : (a) $y=9x^2$ (b) $x = 3$

JANJANG



PROGRESSIONS



TOPIK 5 (TINGKATAN 4)
JANJANG
PROGRESSIONS

- 1 Diberi bahawa 11 , $y + 4$ dan $3y - x$ adalah tiga sebutan berturutan bagi suatu janjang aritmetik.

It is given that 11 , $y + 4$ and $3y - x$ are three consecutive terms of an arithmetic progression.

- (a) Ungkapkan y dalam sebutan x

Express y in terms of x

- (b) Cari beza sepunya jika $x = 8$

Find the common difference if $x = 8$

Jawapan : (a) $y = x - 3$ (b) $- 2$

- 2 Tiga sebutan pertama bagi suatu janjang aritmetik ialah h , 8 dan k . Cari nilai $h + k$.

The first three terms of an arithmetic progressions are h , 8 and k .Find the value of $h + k$.

Jawapan : 16

- 3 Diberi p , $3p - 1$ dan $4p$ adalah tiga sebutan berturut-turut bagi suatu jangjang aritmetik. Cari nilai bagi p .

Given that p , $3p - 1$ and $4p$ are three consecutive terms of an arithmetic progression.

Find the value of p .

Jawapan : $p = 2$

- 4 Sebutan ke- n , T_n suatu jangjang aritmetik diberi sebagai $T_n = 5n - 2$. Cari hasil tambah 7 sebutan pertama jangjang itu.

The n^{th} term , T_n of an arithmetic progression is given as $T_n = 5n - 2$. Find the sum of the first 7 terms of the progression.

Jawapan : 126

- 5 Diberi hasil tambah n sebutan pertama bagi jangjang aritmetik 25 , 18 , 11, ialah 4. Cari nilai bagi n .

Given the sum of the first n term of the arithmetic progression 25 , 18 , 11, is 4.

Find the value of n .

Jawapan : $n = 8$

- 6 Dalam satu jangjang aritmetik , sebutan ke-5 ialah 13 dan sebutan ke-12 ialah 27 . Cari sebutan pertama dan beza sepunya.

In arithmetic progression , the 5th term is 13 and the 12th term is 27. Find the first term and the common difference.

Jawapan : $a = 5 , d = 2$

- 7 Hasil tambah n sebutan pertama suatu jajang aritmetik diberi oleh $S_n = \frac{5}{2}n^2 - 9n$.

Hitung sebutan ke-5.

The sum of the first n term of an arithmetic progression is given by $S_n = \frac{5}{2}n^2 - 9n$.

Calculate the 5th term.

Jawapan : $\frac{27}{2}$

- 8 Hasil tambah n sebutan pertama suatu jajang aritmetik diberi oleh

$S_n = \frac{n}{2}(15 - 5n)$. Hitung sebutan pertama dan beza sepunya.

The sum of the first n term of an arithmetic progression is given by

$S_n = \frac{n}{2}(15 - 5n)$. *Calculate the first term and the common difference.*

Jawapan : $a=5$, $d = -5$

- 9 Seutas tali dengan panjang 45 m dipotong kepada beberapa bahagian supaya panjang setiap bahagian membentuk suatu jangjang aritmetik . Panjang tali yang terpendek dan terpanjang masing-masing ialah 50 cm dan 4 m. Cari bilangan bahagian tali yang telah dipotong.

A piece of string with length 45 m is cut into several pieces such that the length of the pieces forms arithmetic progression. The length of the shortest piece and the longest piece of string is 50 cm and 4 m respectively. Find the number of pieces of strings that are cut.

Jawapan : 20

- 10 Dalam suatu jangjang aritmetik, beza sepunya ialah -5 . Diberi hasil tambah 10 sebutan pertama jangjang itu ialah 45, cari

In an arithmetic progression, the common difference is -5 . Given the sum of the first 10 terms of the progression is 45, find

- (a) sebutan pertama jangjang itu,
the first term of the progression,
- (b) sebutan kesepuluh jangjang itu
the tenth term of the progression

Jawapan : (a) 27 (b) -18

- 11 Tiga sebutan pertama yang positif bagi suatu jangjang geometri ialah 2 , p dan 18. Cari nilai p dan nisbah sepunya bagi jangjang itu.

The first three positive terms of a geometric progression are 2 , p and 18. Find the value of p and the common ratio of the progression.

Jawapan : $p = 6$, $r = 3$

- 12 Diberi bahawa $(x + 1)$, $(2x - 7)$ and $\left(\frac{x+1}{4}\right)$ ialah tiga sebutan berturutan bagi suatu jangjang geometri dengan nisbah sepunya $\frac{1}{2}$.

It is given that $(x + 1)$, $(2x - 7)$ and $\left(\frac{x+1}{4}\right)$ are three consecutive terms of a geometric progression with a common ratio of $\frac{1}{2}$.

Cari / Find

- (a) nilai x ,

the value of x ,

- (b) sebutan pertama jika $(x + 1)$ ialah sebutan ke-12 jangjang itu.

the first term if $(x + 1)$ is the 12th term of the progression

Jawapan : (a) $x = 5$ (b) 12 288

- 13 Dalam suatu jangjang geometri , sebutan pertama ialah a dan nisbah sepunya ialah r .
Diberi sebutan ketiga jangjang itu melebihi sebutan kedua sebanyak $12a$, cari nilai-nilai r .

In a geometric progression , the first term is a and the common ratio is r . Given that the third term of the progression exceeds the second term by $12a$, find the value of r .

Jawapan : -3 , 4

- 14 Maklumat berikut merujuk kepada hasil tambah sebutan-sebutan suatu jangjang geometri.

The following information refers to the sum of the terms of a geometric progression.

$$0.363636... = 0.36 + v + w + ...$$

dengan keadaan v dan w ialah pemalar.

where v and w are constants.

Tentukan / *Determine*

- (a) nilai v dan nilai w ,

the value of v and of w ,

- (b) nisbah sepunya jangjang itu.

the common ratio of the progression.

Jawapan : (a) $v = 0.0036$, $w = 0.000036$ (b) 0.01

- 15 Sebutan ke-10 bagi suatu jangjang aritmetik ialah - 41 dan hasil tambah 10 sebutan pertama jangjang itu ialah -95. Cari

The 10th term of an arithmetic progression is - 41 and the sum of the first 10 terms of the progression is - 95. Find

- (a) sebutan pertama dan beza sepunya
the first term and the common difference
- (b) sebutan negatif pertama bagi jangjang aritmetik itu.
the first negative term of the arithmetic progression

Jawapan : (a) $a = 22$, $d = -7$ (b) -6

- 16 Ungkapkan perpuluhan berulang 0.56363... dalam bentuk hasil tambah ketakterhinggaan bagi suatu jangjang geometri. Seterusnya, ungkapkan nombor itu dalam pecahan termudah.

Express the repeating decimals 0.56363... in the form of sum to infinity of geometric progressions. Hence, express the number in the simplest form of fraction.

Jawapan : $\frac{31}{55}$

17 Diberi $S_n = 2n^2 - 5n$, cari

Given that $S_n = 2n^2 - 5n$, find

(a) sebutan pertama,

the first term,

(b) sebutan ke-9,

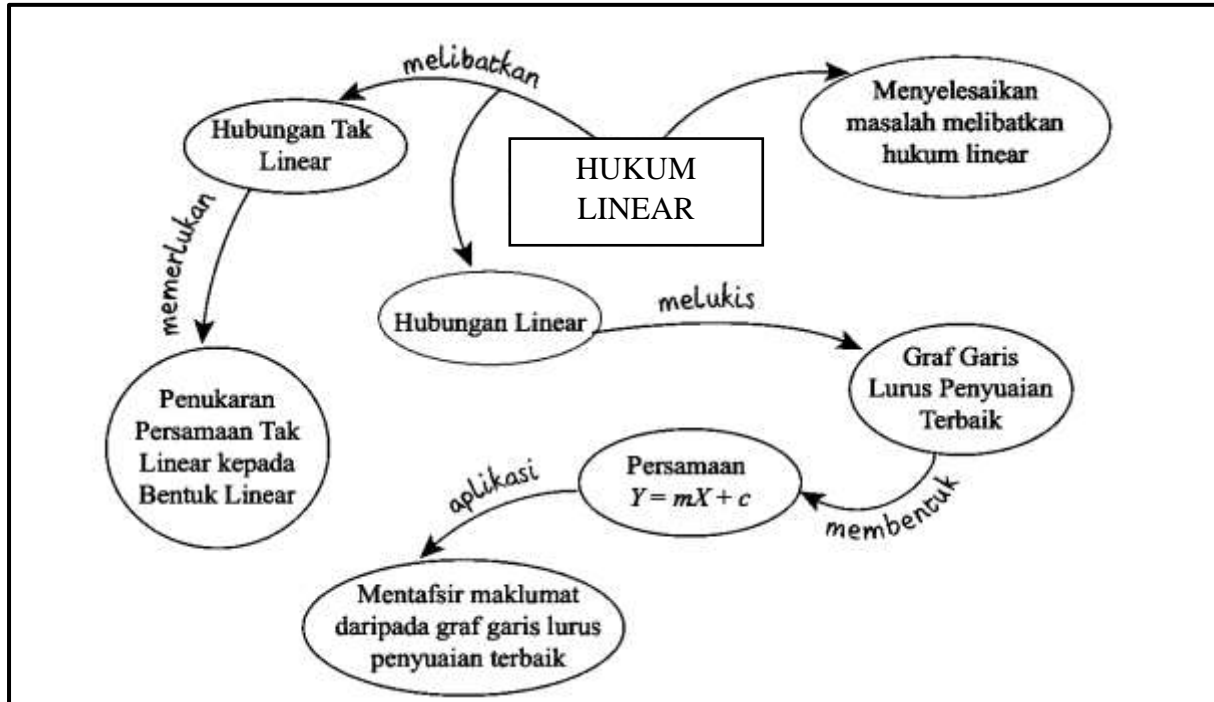
the 9th term,

(c) hasil tambah dari sebutan ke-4 hingga sebutan ke-8.

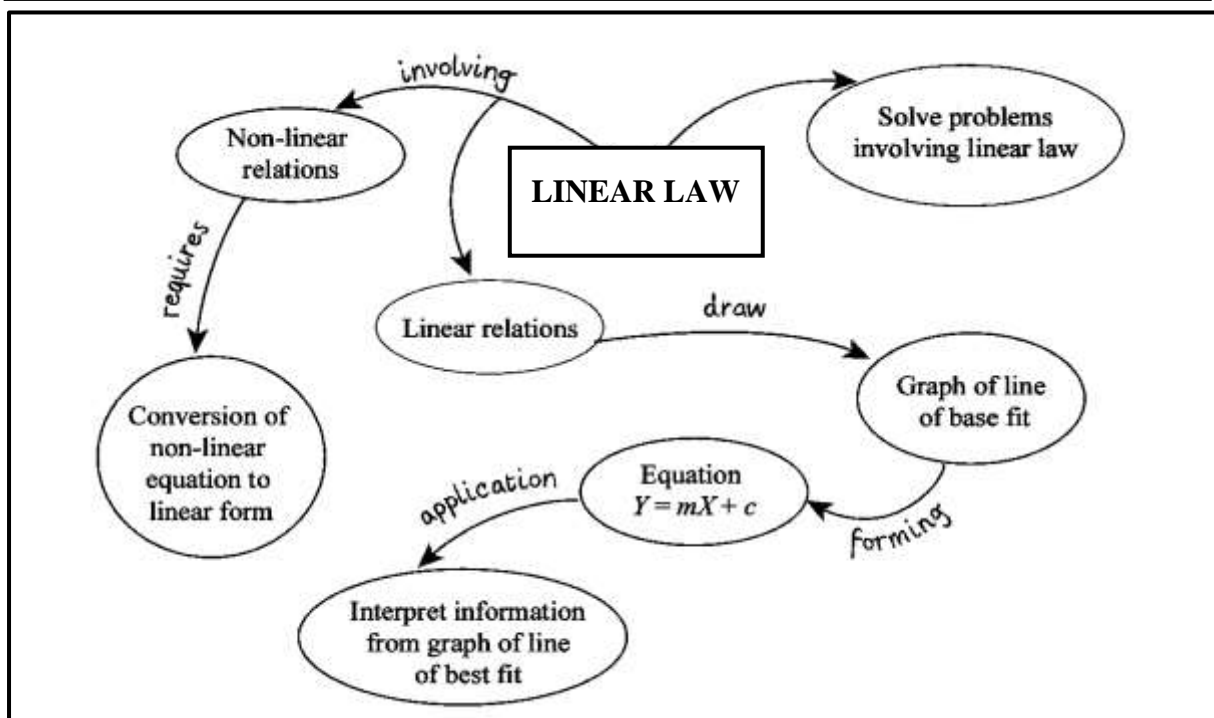
the sum of the 4th term to the 8th term.

Jawapan : (a) – 3 (b) 29 (c) 85

HUKUM LINEAR

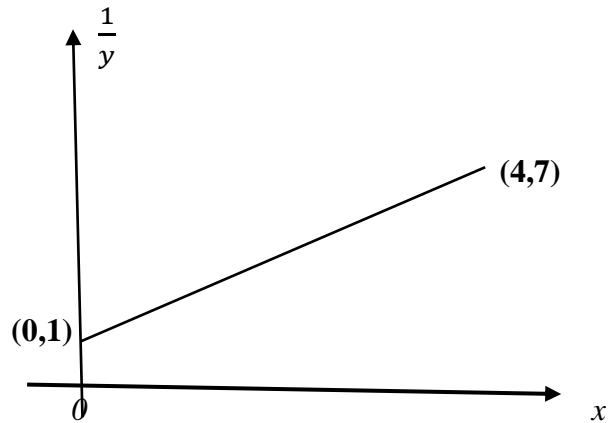


LINEAR LAW



TOPIK 6 (TINGKATAN 4)
HUKUM LINEAR / LINEAR LAW

1.



Rajah 1/ Diagram 1

Rajah 1 menunjukkan satu graf garis lurus yang diperolehi dengan memplot $\frac{1}{y}$ melawan x .

The diagram 1 shows a straight line graph obtained by plotting $\frac{1}{y}$ against x .

(a) Ungkapkan y dalam sebutan x .

Express y in terms of x .

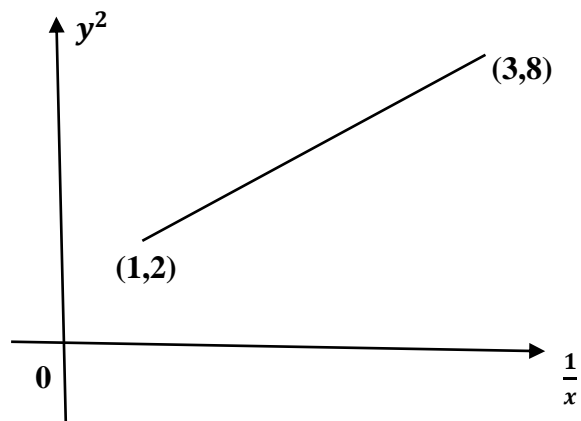
(b) Cari nilai y apabila $x = 6$.

Find the value of y when $x = 6$.

[5 markah/5 marks]

Jawapan/Answer:

2.



Rajah 2/ Diagram 2

Rajah 2 menunjukkan satu graf garis lurus y^2 melawan $\frac{1}{x}$.

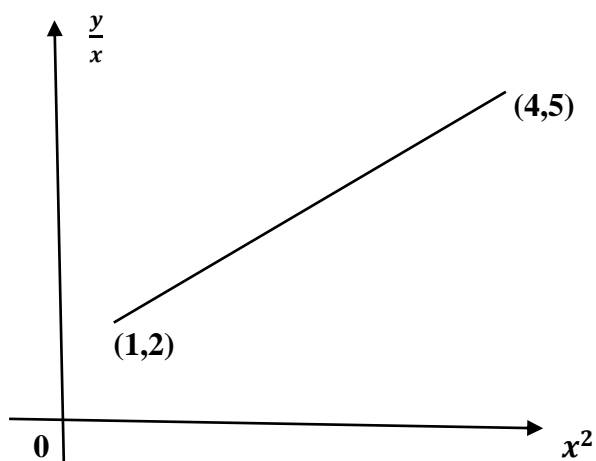
The diagram 2 shows a straight line graph of y^2 against $\frac{1}{x}$.

- Ungkapkan y dalam sebutan x
Express y in terms of x .
- Cari nilai x apabila $y = 4$
Find the value of x when $y = 4$.

[5 markah/5 marks]

Jawapan/Answer:

3.



Rajah 3/ Diagram 3

Rajah 3 menunjukkan satu graf garis lurus $\frac{y}{x}$ melawan x^2

The diagram 3 shows a straight line graph of $\frac{y}{x}$ against x^2 .

(a) Cari hubungan antara $\frac{y}{x}$ dan x^2 .

Find the relation between $\frac{y}{x}$ and x^2 . [4 markah/4 marks]

(b) Cari hubungan antara y dan x .

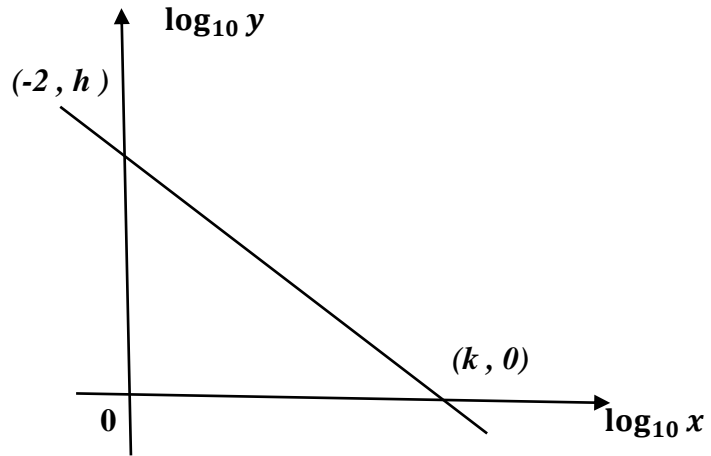
Find the relation between y and x . [2 markah/2 marks]

(c) Cari nilai y apabila $x = 3$

Find the value of y when $x = 3$ [2 markah/2 marks]

Jawapan/Answer:

4.



Rajah 4/ Diagram 4

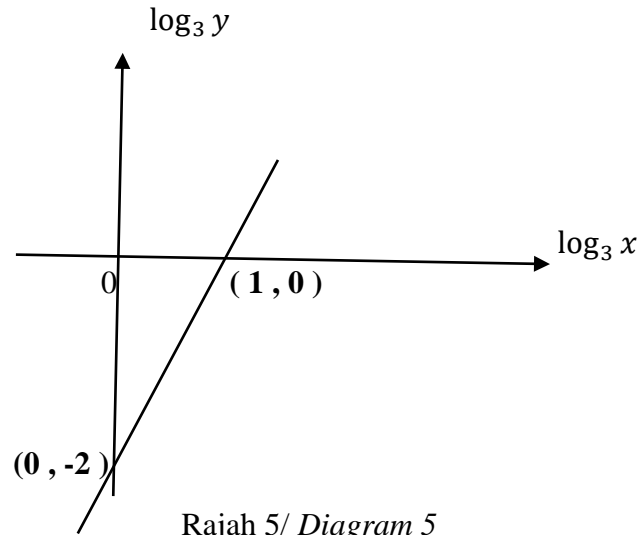
Rajah 4 menunjukkan satu graf garis lurus $y = 100x^{-1/2}$ yang diperoleh dengan memplot $\log_{10} y$ melawan $\log_{10} x$. Cari nilai h dan nilai k .

The diagram 4 shows a straight line graph of $y = 100x^{-1/2}$ obtained by plotting $\log_{10} y$ against $\log_{10} x$. Find the values of h and k .

[5 markah/5 marks]

Jawapan/Answer:

5.



Rajah 5 menunjukkan satu graf garis lurus $\log_3 y$ melawan $\log_3 x$. Pemboleh ubah x dan y dihubungkan oleh persamaan $y = \frac{x^{2n}}{k}$ dengan keadaan n dan k ialah pemalar. Cari nilai n dan nilai k .

The diagram 5 shows a straight line graph of $\log_3 y$ against $\log_3 x$. The variables x and y are related by the equation $y = \frac{x^{2n}}{k}$, where n and k are constants. Find the values of n and k .

[5 markah/5 marks]

Jawapan/Answer:

6. Jadual 1 menunjukkan nilai-nilai dua pemboleh ubah , x^2 dan xy yang diperoleh daripada satu eksperimen.

The table 1 shows the values of two variables , x^2 and xy , obtained from an experiment.

Jadual 1/Table 1

x^2	5	9	16	25	36	42
xy	12	15.5	22	30	40	45

(a) Plot graf xy melawan x^2 , dengan menggunakan skala 2 cm kepada 5 unit pada paksi -X dan paksi-Y. Seterusnya, lukis garis lurus penyuaian terbaik.

Plot xy against x^2 , using a scale of 2 cm to 5 unit on the X-axis and Y-axis. Hence, draw the line of best fit.

[4 markah/4 marks]

(b) Gunakan graf di (a) , untuk mencari,

Use the graph in (a) to find,

(i) kecerunan /gradient

(ii) pintasan-Y/ Y-intercept

(iii) nilai x^2 apabila $xy = 16.5$
value of x^2 when $xy = 16.5$

(iii) nilai y apabila $x = 2.5$
value of y when $x = 2.5$

(c) Hitung nilai x apabila $xy = 100$

Calculate the value of x when $xy = 100$

[6 markah/6 marks]

7. Jadual 2 menunjukkan nilai-nilai dua pemboleh ubah , $\log_{10}(x + 1)$ dan $\log_{10} y$ yang diperoleh daripada satu eksperimen.

The table 2 shows the values of two variables , $\log_{10}(x + 1)$ and $\log_{10} y$, obtained from an experiment.

Jadual 2/ Table 2

$\log_{10}(x + 1)$	0.18	0.30	0.50	0.60	0.70	0.78
$\log_{10} y$	0.33	0.45	0.64	0.75	0.85	0.93

(a) Plot graf $\log_{10} y$ melawan $\log_{10}(x + 1)$, dengan menggunakan skala 2 cm kepada 0.1 unit pada paksi $\log_{10}(x + 1)$ and paksi $\log_{10} y$. Seterusnya, lukis garis lurus penyesuaian terbaik.

Plot $\log_{10} y$ against $\log_{10}(x + 1)$, using a scale of 2 cm to 0.1 unit on the $\log_{10}(x + 1)$ - axis and $\log_{10} y$ - axis. Hence, draw the line of best fit.

[4 markah/4 marks]

(b) Gunakan graf di (a) , untuk mencari,

Use the graph in (a) to find

- (i) kecerunan / *gradient*
- (ii) pintasan- $\log_{10} y$
 $\log_{10} y$ - intercept
- (iii) nilai x apabila $\log_{10} y = 0.55$
value of x when $\log_{10} y = 0.55$

(c) Hitung / *Calculate*

- (i) nilai y apabila $x = 2.5$
the value of y when $x = 2.5$
- (ii) nilai x apabila $y = 1.5$
the value of x when $y = 1.5$

[6 markah/6 marks]

8. Jadual di bawah menunjukkan nilai-nilai eksperimen bagi pemboleh ubah x dan y , yang dihubungkan oleh persamaan $\frac{a}{x} + \frac{b}{y} = 1$, dengan keadaan a dan b ialah pemalar.

The table shows the experimental values of the variables x dan y , which are related by the equation $\frac{a}{x} + \frac{b}{y} = 1$, where a dan b are constants.

x	1	0.5	0.25	0.20	0.125
y	0.8	0.5	0.29	0.24	0.155

- (a) Plotkan graf $\frac{1}{y}$ melawan $\frac{1}{x}$
Plot the graph of $\frac{1}{y}$ against $\frac{1}{x}$ [4 markah/4 marks]
- (b) Berdasarkan graf itu, cari nilai a dan nilai b
Based on the graph, find the values of a and b . [4 markah/4 marks]
- (c) Cari nilai y apabila $x = 0.3$
Find the value of y when $x = 0.3$. [2 markah/2 marks]

9. Jadual di bawah menunjukkan nilai-nilai dua pemboleh ubah x dan y , yang diperolehi daripada satu eksperimen. Pemboleh ubah x dan y dihubungkan oleh persamaan

$$y = \frac{p}{x} + \frac{q}{x^2}, \text{ dengan keadaan } p \text{ dan } q \text{ ialah pemalar.}$$

The table shows the values of two variables x and y , obtained from an experiment. The variables x and y are related by the equation $y = \frac{p}{x} + \frac{q}{x^2}$, where p and q are constants.

x	1.0	1.5	2.0	2.5	3.0	3.5
y	0.8	5.20	4.95	4.48	4.00	3.57

- (a) Plot xy melawan $\frac{1}{x}$, dengan menggunakan skala 2 cm kepada 0.2 unit pada paksi- $\frac{1}{x}$ dan 2 cm kepada 2 unit pada paksi- xy . Seterusnya, lukis garis lurus penyuaian terbaik.

Plot xy against $\frac{1}{x}$, using a scale of 2 cm to 0.2 unit on the $\frac{1}{x}$ -axis and 2 cm to 2 unit on the xy -axis. Hence, draw the line of best fit.

[4 markah/4 marks]

- (b) Gunakan graf anda di (a), untuk mencari,
Use your graph in (a) to find

(i) nilai p dan nilai q
the values of p and of q

(ii) nilai x apabila $y = \frac{5}{x}$
value of x when $y = \frac{5}{x}$

[6 markah/6 marks]

10. Jadual di bawah menunjukkan nilai-nilai dua pemboleh ubah x dan y , yang diperolehi daripada satu eksperimen. Pemboleh ubah x dan y dihubungkan oleh persamaan $y = (k + 1)x^n$, dengan keadaan k dan n ialah pemalar.
The table shows the values of two variables x and y , obtained from an experiment. The variables x and y are related by the equation $y = (k + 1)x^n$, where k and n are constants.

x	5	10	15	20	25	30
y	56	177	355	563	794	1122

- (a) Plot $\log_{10} y$ melawan $\log_{10} x$, dengan menggunakan skala 2 cm kepada 0.2 unit pada paksi- $\log_{10} x$ dan 2 cm kepada 0.5 unit pada paksi- $\log_{10} y$. Seterusnya, lukis garis lurus penyuaian terbaik.
Plot $\log_{10} y$ against $\log_{10} x$, by using a scale of 2 cm to 0.2 unit on the $\log_{10} x$ - axis and 2 cm to 0.5 unit on the $\log_{10} y$ -axis. Hence, draw the line of best fit.

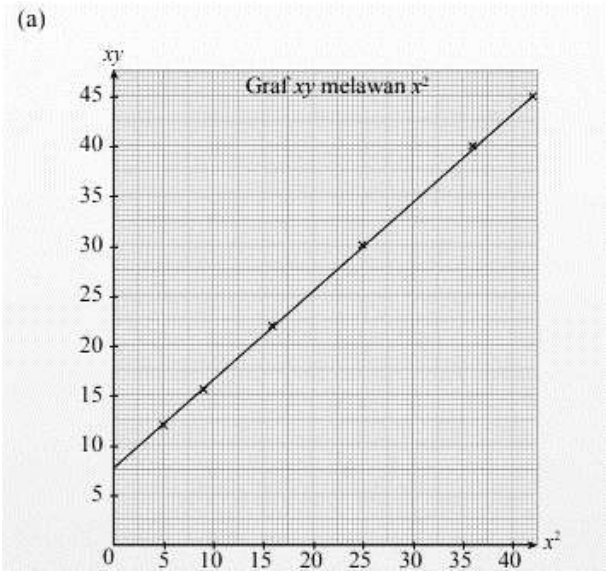
[5 markah/5 marks]

- (b) Gunakan graf anda di (a), untuk mencari,
Use your graph in (a) to find

- (i) nilai k dan nilai n
the values of k and n
- (ii) nilai x apabila $y = 126$
value of x when $y = 126$

[5 markah/5 marks]

SKEMA JAWAPAN: HUKUM LINEAR

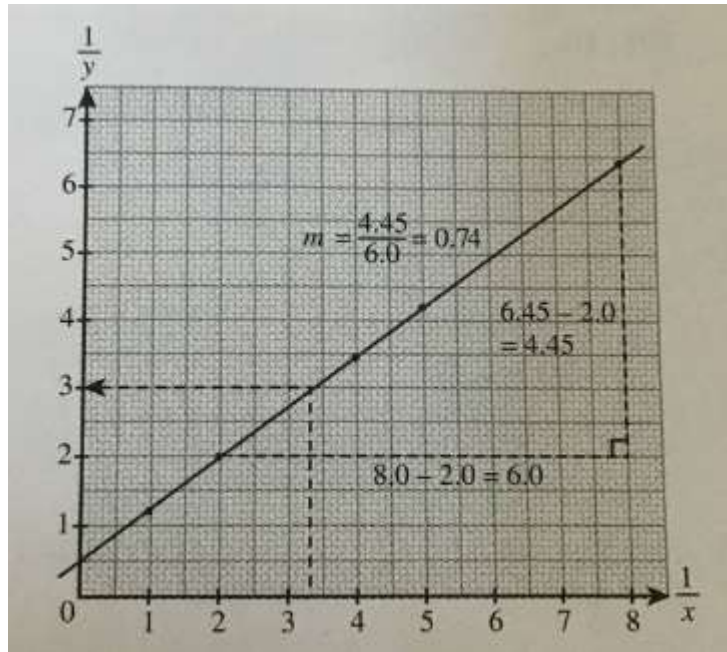
NO SOALAN	JAWAPAN	JUMLAH MARKAH
1	(a) $y = \frac{2}{3x+2}$ (b) $y = \frac{1}{10}$	5
2	(a) $y = \sqrt{\frac{3-x}{x}}$ (b) $x = \frac{3}{17}$	5
3	(a) $\frac{y}{x} = x^2 + 1$ (b) $y = x^3 + x$ (c) $y = 30$	8
4	$h = 3, k = 4$	5
5	$n = 1, k = 9$	5
6	(a) 	

8

(a)

$\frac{1}{x}$	1	2.0	4.0	5.0	8.0
$\frac{1}{y}$	1.25	2.0	3.45	4.17	6.45

4



4

2

(b) $a = -1.48$, $b = 2$ (c) $y = 0.34$

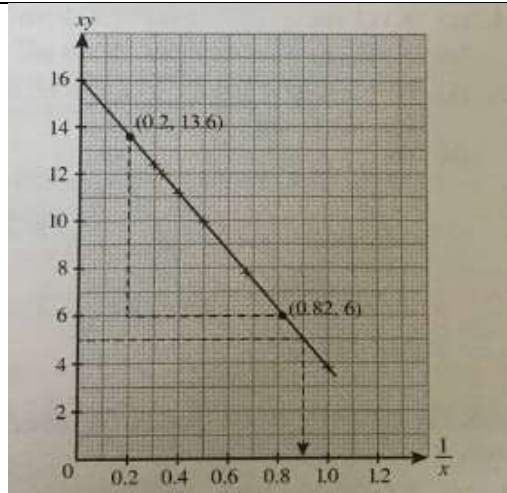
10

9

(a)

$\frac{1}{x}$	1	0.67	0.5	0.4	0.33	0.29
xy	1.25	2.0	3.45	4.17	6.45	12.50

4



(b) (i) $p = 16$, $q = -12.26$

(ii) $x = 1.11$

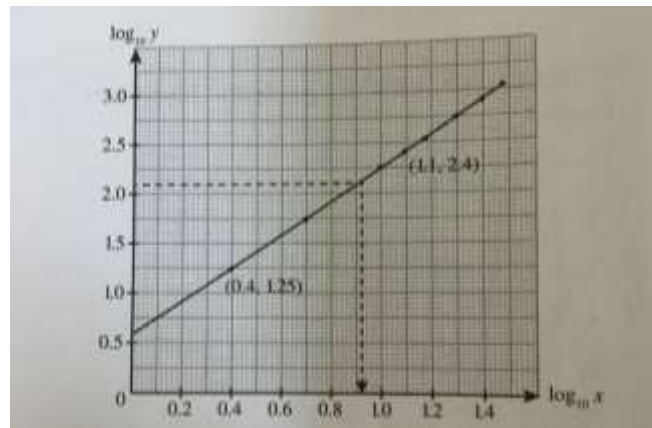
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10

10

(a)

$\log x$	0.70	1.00	1.18	1.30	1.40	1.48
$\log y$	1.75	2.25	2.55	2.75	2.90	3.05



(b) (i) $k = 2.98$, $n = 1.64$

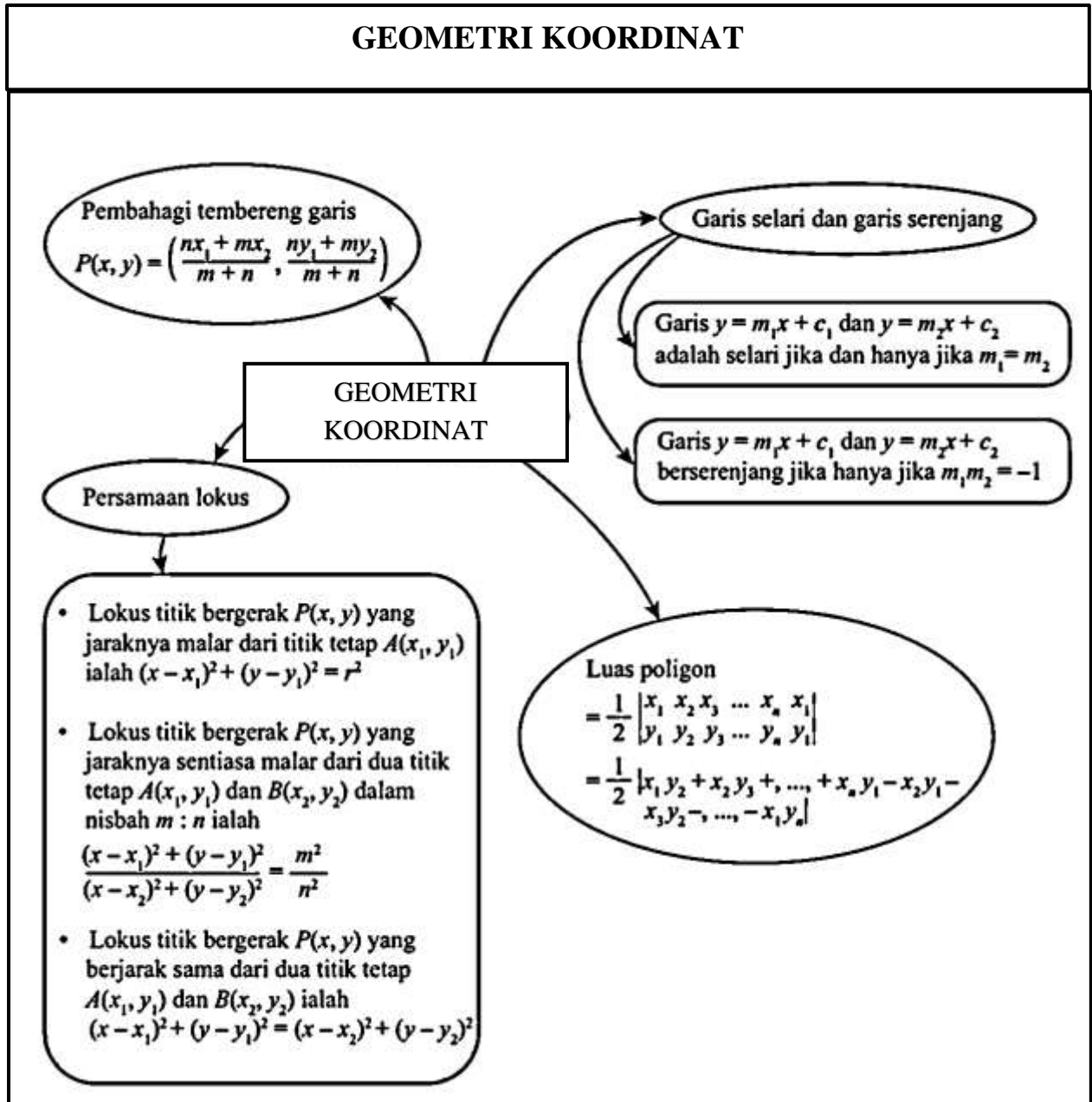
(ii) $x = 8.32$

5

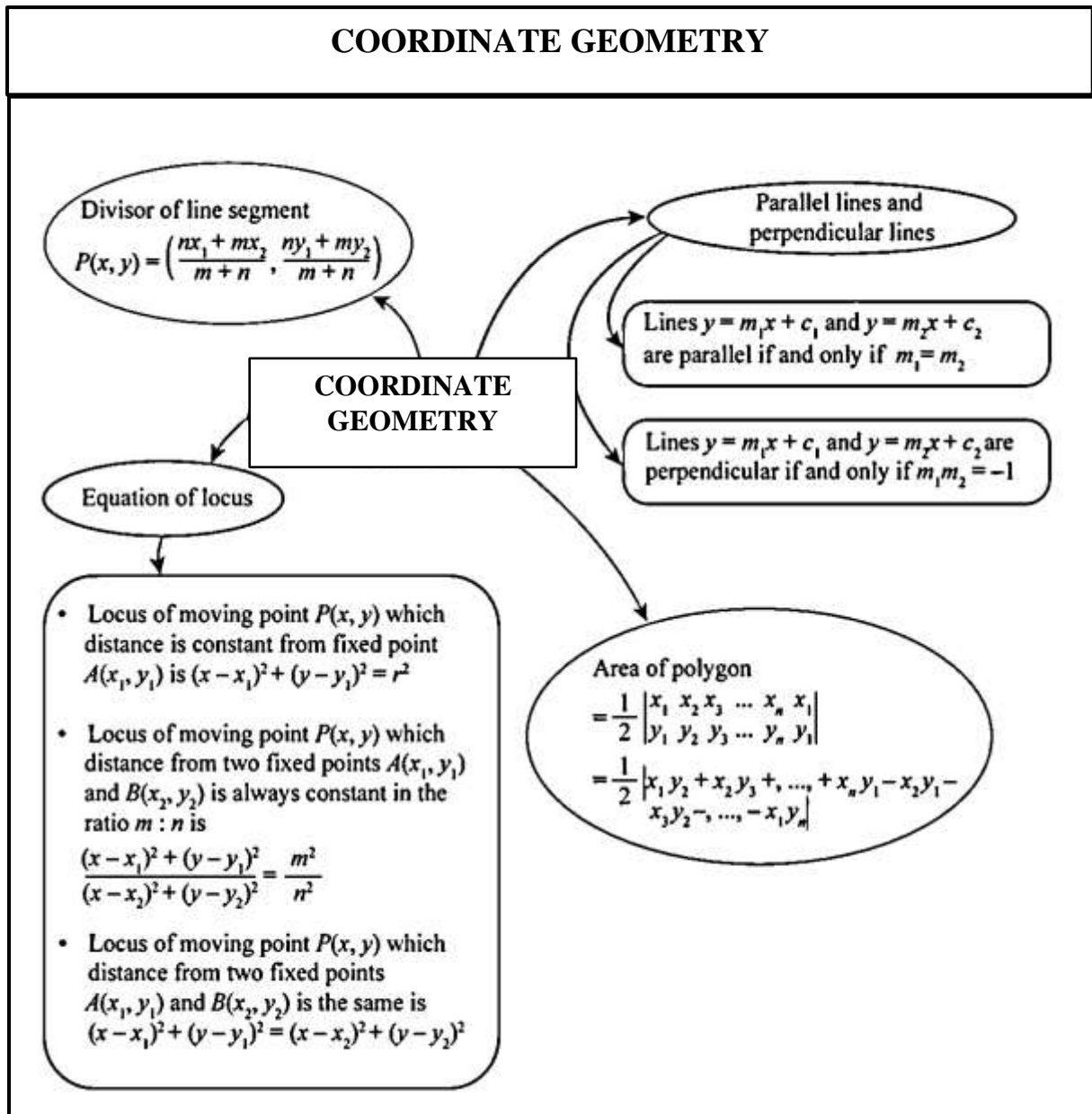
5

10

GEOMETRI KOORDINAT



COORDINATE GEOMETRY



TOPIK 7 TINGKATAN 4

GEOMETRI KOODINAT/ COORDINATE GEOMETRY

1. $E(-5, -8)$, $F(-9, 8)$, $G(7, 2)$, dan H ialah bucu-bucu bagi suatu segi empat selari.
Carikan koordinat bucu H .

$E(-5, -8)$, $F(-9, 8)$, $G(7, 2)$ and H are the vertices of a parallelogram.

Find the coordinates of the point H .

[4 markah]

[4 marks]

Jawapan /Answer : $H(11, -14)$

2. Diberi titik-titik $P(3, -2)$ dan $Q(-1, 6)$. Carikan persamaan pembahagi dua sama seranjang bagi garis lurus PQ .

Given points $P(3, -2)$ and $Q(-1, 6)$. Find the equation of the perpendicular bisectors of the straight line PQ .

[4 markah]

[4 marks]

Jawapan /Answer : $x-2y+3=0$

3. a) Carikan titik persilangan bagi garis-garis lurus $2x - 2y = 7$ dan $8x - 7y = 5$.
Find the point of intersection of the straight lines $2x - 2y = 7$ and $8x - 7y = 5$.

[3 markah]

[3 marks]

- b) Titik $E(-4, 9)$ membahagikan garis lurus yang menyambungkan $P(-7, 8)$ dan $Q(8, 13)$ dalam nisbah $a : b$. Carikan nilai a dan b .
The point $E(-4, 9)$ divides a straight line joining $P(-7, 8)$ and $Q(8, 13)$ in the ratio $a : b$. Find the values of a and of b .

[3 markah]

[3 marks]

Jawapan/Answer: (a) $(-19\frac{1}{2}, -23)$, (b) $a=1$ $b=4$

4. a) Titik $K(e, f)$ membahagikan garis lurus yang menyambungkan $J(-12g, -5g)$ dan $M(5e, -5f)$ dalam nisbah $2 : 3$. Ungkapkan e dalam sebutan f .
The point $K(e, f)$ divides a straight line joining $J(-12g, -5g)$ and $M(5e, -5f)$ in the ratio $2 : 3$. Express e in terms of f .

[4 markah]

[4 marks]

- b) Carikan persamaan lokus bagi titik bergerak R di mana jaraknya dari $P(-4, 7)$ dan $Q(-1, -10)$ adalah sama.
Find the equation of the locus of moving point R such that its distances from $P(-4, 7)$ and $Q(-1, -10)$ are equal.

[4 markah]

[4 marks]

Jawapan/Answer : (a) $e = -\frac{36}{5}f$ (b) $3x-17y-18=0$

5. a) Tunjukkan bahawa titik-titik $E(-6, -6)$, $F(-4, -4)$, dan $G(2, 2)$ adalah segaris.
Show that points $E(-6, -6)$, $F(-4, -4)$ and $G(2, 2)$ are collinear.

[3 markah]

[3 marks]

- b) Carikan persamaan lokus bagi titik bergerak R di mana jaraknya dari $M(9, 2)$ dan $N(10, -3)$ adalah dalam nisbah 1 : 3.

Find the equation of the locus of moving point R such that its distances from $M(9, 2)$ and $N(10, -3)$ are in the ratio 1 : 3.

[3 markah]

[3 marks]

Jawapan/Answer: (b) $4x^2+4y^2-71x-21y+328=0$

6. Locus titik bergerak M diberi sebagai $x^2 + y^2 + 12x + y + 35 = 0$.
The locus of a moving point M is given by $x^2 + y^2 + 12x + y + 35 = 0$.
Tunjukkan bahawa
Show that

(a) locus M bersilang dengan paksi- x pada dua titik.
the locus M intersects the x -axis at two points.

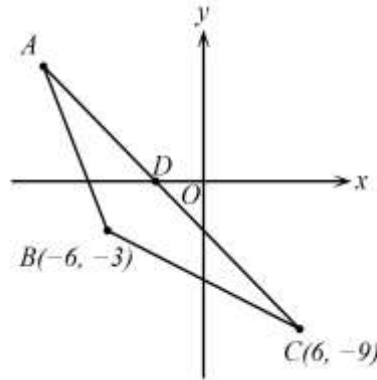
[4 markah]
[4 marks]

(b) garis lurus $y = -2x - 10$ adalah tangen kepada locus M .
the straight line $y = -2x - 10$ is a tangent to the locus M .

[4 markah]
[4 marks]

7. Rajah 7 menunjukkan segi tiga ABC dengan luas 48 unit^2 . Diberi $B(-6, -3)$ dan $C(6, -9)$.
 Persamaan garis lurus AC ialah $x + y + 3 = 0$. Titik D terletak pada paksi- x dan
 membahagikan garis lurus AC dalam nisbah $p : q$.

*Diagram 7 shows a triangle ABC with an area of 48 unit^2 . Given $B(-6, -3)$ and $C(6, -9)$.
 The equation of the straight line AC is $x + y + 3 = 0$. The point D lies on the x -axis and
 divides the straight line AC in the ratio $p : q$.*



Rajah 7
Diagram 7

Carikan

Find

- (a) koordinat titik A ,
the coordinates of the point A ,

[8 markah]

[8 marks]

- (b) $p : q$

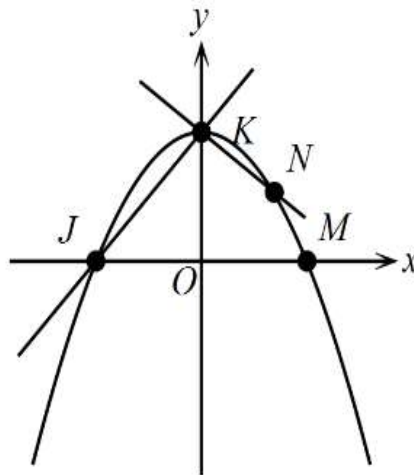
[2 markah]

[2 marks]

Jawapan /Answer : (a) $A(-10, 7)$ (b) $7:9$

8. Rajah 8 menunjukkan lengkung $y = -5x^2 + 20$ yang bersilang dengan paksi- x pada titik J dan M , dan bersilang dengan paksi- y pada titik K . Garis lurus KN adalah berserenjang dengan garis lurus JK dan bersilang dengan lengkung itu pada titik N .

Diagram 8 shows the curve $y = -5x^2 + 20$ which cuts the x -axis at the point J and M , and cuts the y -axis at point K . The straight line KN , which is perpendicular to the straight line JK , cuts the curve at the point N .



Rajah 8
Diagram 8

Carikan

Find

- (a) persamaan garis lurus JK ,
the equation of the straight line JK ,

[3 markah]

[3 marks]

- (b) persamaan garis lurus KM ,
the equation of the straight line KM ,

[2 markah]

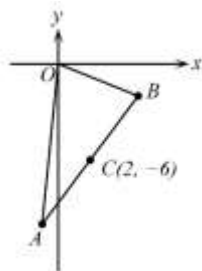
[2 marks]

- (c) koordinat titik N .
the coordinates of the point N .

[5 markah]
 [5 marks]

Jawapan / Answer : (a) $y = 10x + 20$ (b) $y = -10x + 20$, (c) $N(\frac{1}{50}, 19\frac{499}{500})$

9. Rajah 9 menunjukkan segi tiga OAB . Titik $C(2, -6)$ terletak pada garis lurus AB . Koordinat titik B ialah $(5, e)$.
Diagram 9 shows a triangle OAB . Point $C(2, -6)$ lies on the straight line AB . The coordinates of point B is $(5, e)$.



Rajah 9
Diagram 9

- (a) Carikan persamaan lokus bagi titik bergerak P di mana jaraknya dari C sentiasa 5 unit.
Find the equation of the locus of a moving point P such that its distance from C is always 5 units.

[2 markah]
 [2 marks]

- (b) Diberi titik A dan titik B terletak pada lokus P . Kirakan
Given that point A and point B lie on the locus of P . Calculate
- (i) nilai e ,
the value of e ,
 - (ii) koordinat A .
the coordinates of A .

[6 markah]

[6 marks]

- (c) Seterusnya, carikan luas, dalam unit^2 , segi tiga OAB .
Hence, find the area, in unit^2 , of triangle OAB .

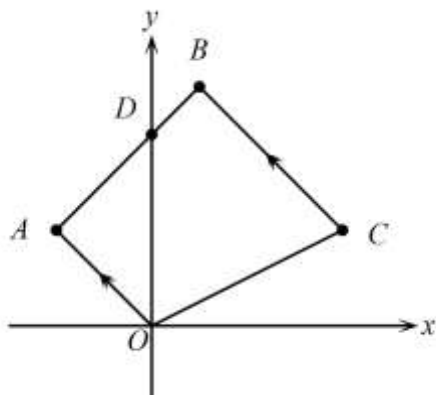
[2 markah]

[2 marks]

Jawapan/Answer : (a) $x^2 + y^2 - 4x + 12y + 15 = 0$, (b) (i) $e = -2$ (ii) $A(-1, -10)$, (c) 26unit^2

10. Rajah 10 menunjukkan trapezium $OABC$. Garis lurus OA adalah berserenjang dengan garis lurus AB yang bersilang dengan paksi- y pada titik D . Diberi persamaan OA ialah $y = -x$ dan persamaan AB ialah $2y = ex + 16$.

Diagram 10 shows a trapezium $OABC$. The line OA is perpendicular to the line AB which intersects with y -axis at the point D . It is given that the equation of OA is $y = -x$ and the equation of AB is $2y = ex + 16$.



Rajah 10
Diagram 10

- (a) Carikan/Find
- (i) nilai e
the value of e ,
 - (ii) koordinat A.
the coordinates of A.

[4 markah]
[4 marks]

(b) Diberi $AD : DB = 2 : 1$, carikan

Given $AD : DB = 2 : 1$, find

(i) koordinat D .

the coordinates of D .

(ii) persamaan garis lurus BC .

the equation of the straight line BC .

[4 markah]

[4 marks]

(c) Titik J bergerak dengan keadaan $JA = 2JB$. Carikan persamaan lokus J .

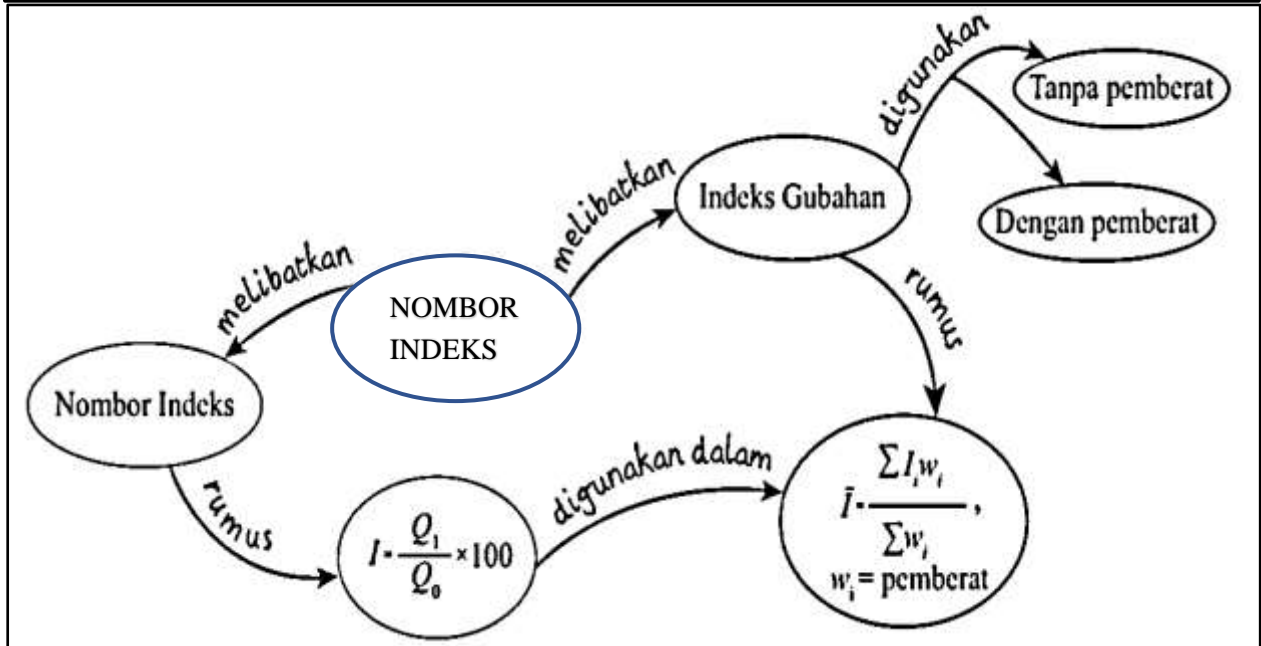
A point J moves such that $JA = 2JB$. Find the equation of the locus of J .

[2 markah]

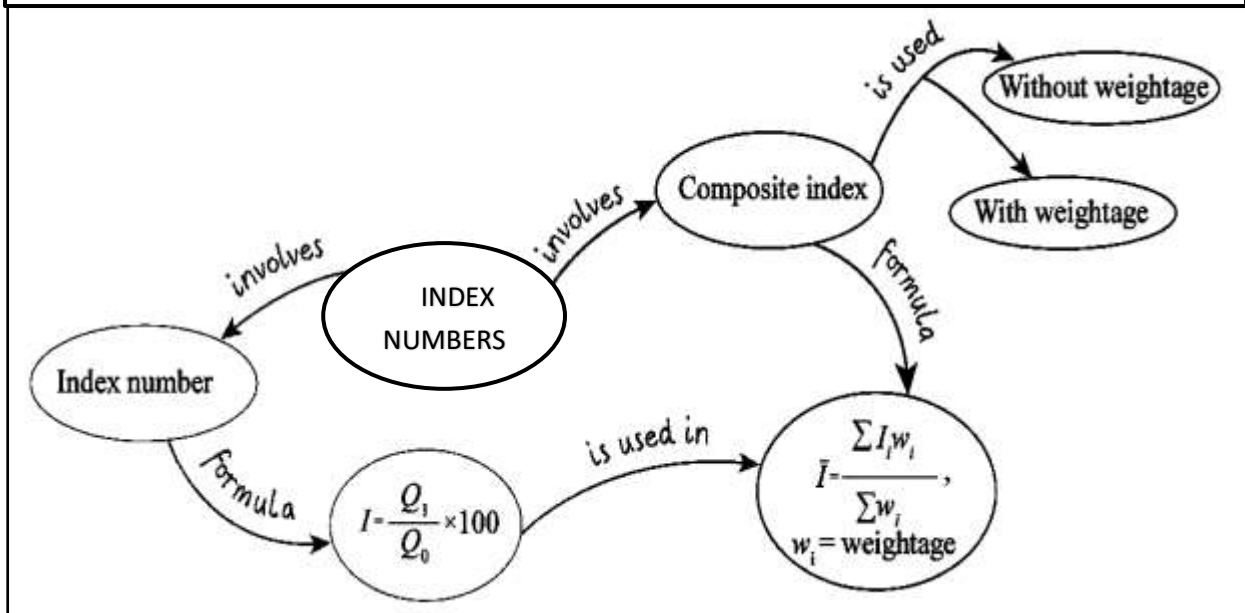
[2 marks]

Jawapan /Answer : a)(i) $e=2$ (ii) $A(-4,4)$, (b)(i) $B(2,10)$ (ii) $y=-x+12$, (c) $x^2+y^2-8x-24y+128=0$

NOMBOR INDEKS



INDEX NUMBER



TOPIK 10 (TINGKATAN 4)

NOMBOR INDEKS / INDEX NUMBER

- 1 Sebuah syarikat mendapat keuntungan RM 40 000 pada tahun 2016 dan RM 58 000 pada tahun 2018. Hitung nombor indeks yang menunjukkan perubahan keuntungan syarikat itu pada tahun 2018 berdasarkan tahun 2016. Seterusnya, tafsirkan nombor indeks yang diperoleh.

A company made a profit of RM 40 000 in the year 2016 and RM 58 000 in the year 2018. Calculate the index number to show the change in profit for the company in the year 2018 based on the year 2016. Hence, interpret the index number obtained.

(Jawapan : 145, peningkatan 45%)

- 2 Bilangan pencetak yang dijual di sebuah kedai pada tahun 2011 dan 2018 masing – masing ialah 320 buah dan 560 buah. Hitung indeks jualan pada tahun 2018 berdasarkan tahun 2011. Seterusnya, tafsirkan indeks jualan yang diperoleh.

The number of printers sold in a shop in the years 2011 and 2018 were 320 and 560 respectively. Calculate the sales index in the year 2018 based on the year 2011. Hence, interpret the sales index obtained.

(Jawapan :175, peningkatan 75%)

- 3 Indeks harga bagi sebuah rumah pada tahun 2019 berdasarkan tahun 2010 dan 2015 masing – masing ialah 150 dan 125. Cari indeks harga bagi rumah tersebut pada tahun 2015 berdasarkan tahun 2010.

The price indices of a house in the year 2019 based on the year 2010 and 2015 were 150 and 125 respectively. Find the price index of the car in the year 2015 based on the year 2010

(Jawapan : 120)

- 4 Nombor indeks yang menunjukkan perubahan bilangan murid sekolah rendah yang memakai cermin mata pada tahun 2014 berasaskan tahun 2010 dan 2012 masing – masing ialah 161 dan 140. Cari nombor indeks bagi bilangan murid pada tahun 2012 berasaskan tahun 2010.

The index numbers that show the change in the number of primary students who wore spectacles in the year 2014 based on the year 2010 and 2012 were 161 and 140 respectively. Find the index number of the number of students in the year 2012 based on the year 2010

(Jawapan : 115)

- 5 Satu kajian mengenai bilangan kemalangan jalan raya telah dijalankan. Bilangan kemalangan jalan raya pada tahun 2010 ialah 13 260. Diberi indeks kemalangan pada tahun 2010 berasaskan tahun 2008 ialah 130. Hitung bilangan kemalangan jalan raya pada tahun 2008.

A survey has been made on the number of road accidents. The number of road accidents in the year 2010 was 13 260. Given the accident index in the year 2010 based on the year 2008 is 130, calculate the number of road accidents in the year 2008.

(Jawapan : 10 200)

- 6 Harga sebuah kereta pada tahun 2016 ialah RM 67 500. Diberi indeks harga bagi kereta itu pada tahun 2016 berasaskan tahun 2014 ialah 125, hitung harga kereta itu pada tahun 2014.

The price of a car in the year 2016 was RM 67 500. Given the price index of the car in the year 2016 based on the year 2014 is 125, calculate the price of the car in the year 2014.

(Jawapan :RM54 000)

- 7 Jadual di bawah menunjukkan harga bagi dua barangan A dan B pada tahun 2015 dan 2018.

The table shows the prices of two items A and B the years 2015 and 2018.

Barang (Items)	Harga (RM) Price (RM)	
	2015	2018
A	3.50	4.90
B	6.00	8.10

Hitung indeks harga bagi A dan B pada tahun 2018 berasaskan tahun 2015.

Calculate the price indices of A and B in the year 2018 based on the year 2015

(Jawapan : A=140, B=135)

- 8 Jadual menunjukkan indeks harga bagi empat jenis bahan pada tahun 2017 berasaskan tahun 2014. Hitung indeks gubahan bagi bahan – bahan tersebut pada tahun 2017 berasaskan tahun 2014.

The table shows the price indices of four types of ingredients in the year 2017 based on the year 2014. Calculate the composite index of the ingredients in the year 2017 based on the year 2014.

Bahan / ingredients	Indeks harga pada tahun 2017 berasaskan tahun 2014 / Price index in the year 2017 based on the year 2014
P	112
Q	136
R	109
S	125

(Jawapan : 120.5)

- 9 Jadual menunjukkan harga bagi empat komponen pada tahun 2019 dan tahun 2020 serta pemberatnya masing – masing. Hitung indeks gubahan pada tahun 2020 berdasarkan tahun 2019.

The table shows the prices of four components in the years 2019 and 2020, and their respective weightage. Calculate the composite index in the year 2020 based on the year 2019

Komponen / Components	Harga / Price (RM)		Pemberat / Weightage
	2019	2020	
A	80	107	5
B	96	120	2
C	120	186	4
D	110	154	2

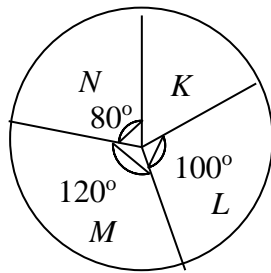
(Jawapan : 139.90)

- 10 Jadual 10 menunjukkan harga dan indeks harga bagi empat bahan, *K*, *L*, *M* dan *N*, yang digunakan untuk membuat sejenis roti. Rajah 10 ialah satu carta pai yang mewakili nisbah kuantiti bahan *K*, *L*, *M* dan *N* yang digunakan untuk membuat roti itu.

Table 10 shows the prices and the price indices for the four ingredients K, L, M, and N, used in making a particular kind of bread. Diagram 10 is a pie chart which represents the ratio of the amount of the ingredients K, L, M and N used in making the bread.

Bahan Ingredients	Harga / Price (RM / kg)		Indeks harga pada tahun 2006 berdasarkan tahun 2003 Price index for the year 2006 based on the year 2003
	Tahun / Year		
	2003	2006	
<i>K</i>	1.60	2.00	<i>p</i>
<i>L</i>	4.00	<i>q</i>	120
<i>M</i>	0.80	1.20	150
<i>N</i>	<i>r</i>	1.60	80

Jadual 10 / Table 10



Rajah 10 / Diagram 10

- (a) Cari nilai p , q dan r .
Find the values of p , q and r .

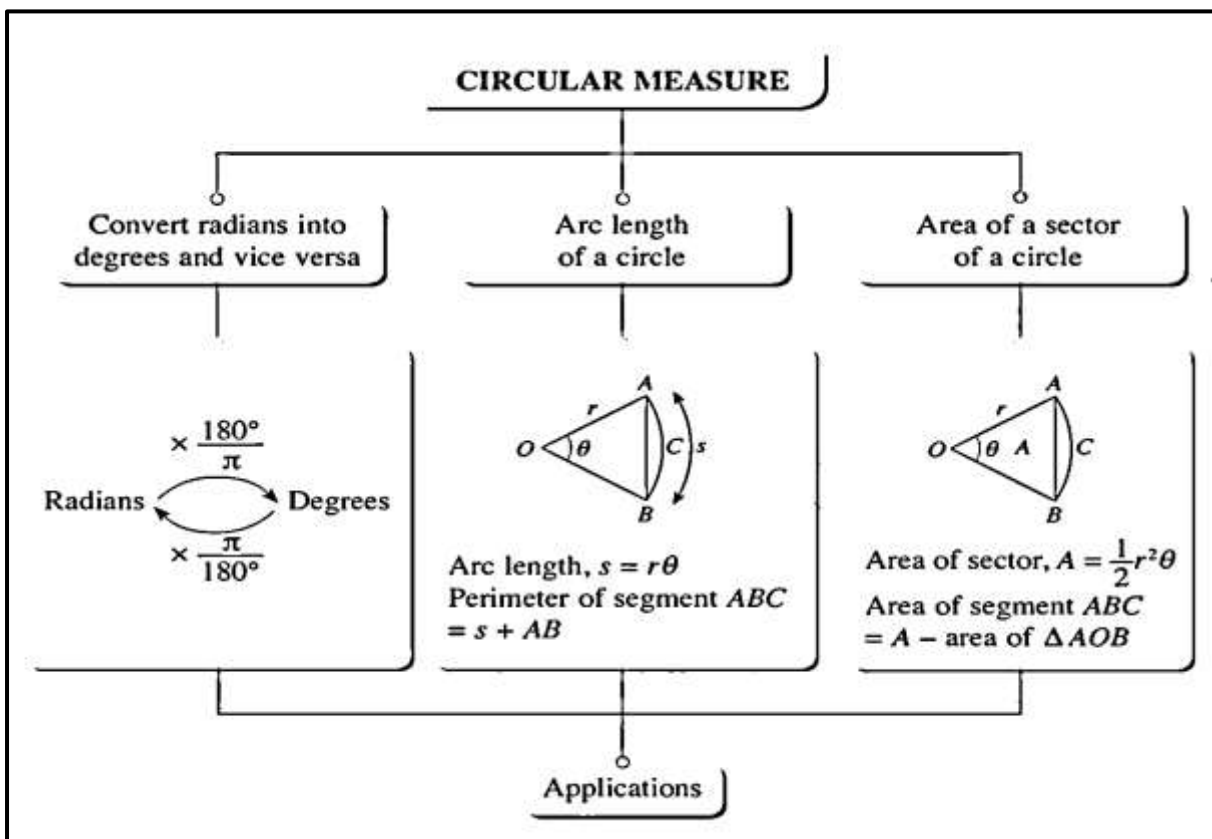
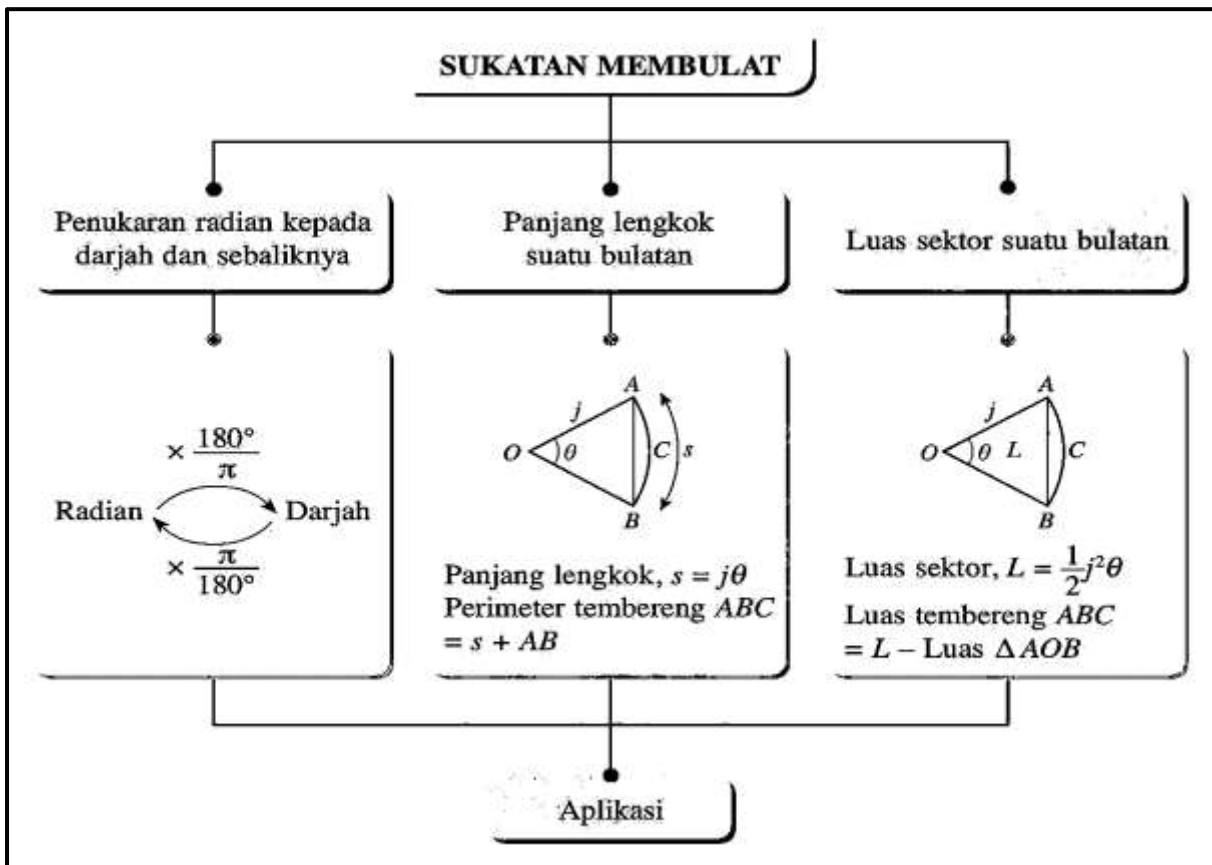
(Jawapan : $p=125$, $q=4.80$, $r=2.00$)

- (b) (i) Hitung indeks gubahan bagi kos untuk membuat roti itu pada tahun 2006 berasaskan tahun 2003.
Calculate the composite index for the cost of making the bread in the year 2006 based on the year 2003.

(Jawapan : 121.94)

- (ii) Oleh yang demikian, hitung kos untuk membuat roti itu pada tahun 2003 jika kos sepadan pada tahun 2006 ialah RM40.
Hence, calculate the cost of making the bread in the year 2003 if the corresponding cost in the year 2006 is RM 40.

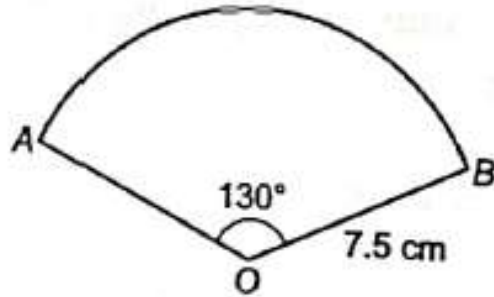
(Jawapan : RM32.80)



BAB 1 (TINGKATAN 5)

SUKATAN MEMBULAT / CIRCULAR MEASURE

1. Rajah menunjukkan sebuah sector AOB berpusat O. Cari
The diagram shows a sector AOB at centre O. Find

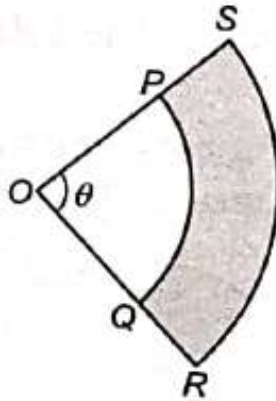


- (a) $\angle AOB$ dalam radian
 $\angle AOB$ in radians
(b) perimeter, dalam cm, sector AOB
the perimeter, in cm, of the sector AOB

[3 markah/marks]

[Jawapan (a) 2.2689 rad (b) 32.02cm]

2. Rajah menunjukkan dua sector, POQ dan SOR, berpusat O.
The diagram shows two sectors, POQ and SOR, with center O.



Diberi bahawa $OP = 6\text{cm}$, nisbah $OP : OS = 2 : 3$ dan luas rantau berlerek ialah 33.75 cm^2 , cari

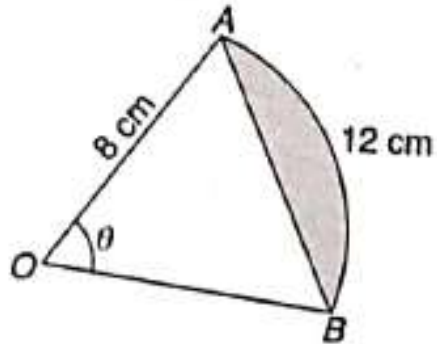
Given that $OP = 6\text{cm}$, the ratio of $OP : OS = 2 : 3$ and the area of the shaded region is 33.75 cm^2 , find

- (a) panjang OS
the length of OS
- (b) nilai θ , dalam radian
the value of θ , in radians

[4 markah/marks]

[Jawapan (a) 9 cm (b) 1.5 rad]

3. Rajah menunjukkan sebuah sector AOB berpusat O dan berjejari 8 cm.
The diagram shows a sector AOB with centre O and a radius of 8 cm.



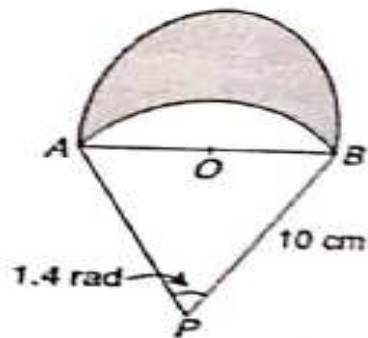
Diberi panjang lengkok AB ialah 12 cm, cari
Given the length of arc AB is 12 cm, find

- (a) sudut θ
the angle θ
(b) panjang perentas AB
the length of the chord AB

[3 markah/marks]

[Jawapan (a) 1.5 rad (b) 10.91 cm]

4. Rajah menunjukkan sebuah semi bulatan berpusat O dan sebuah sector APB dengan pusat P yang berjari 10 cm.
The diagram shows a semicircle with centre O and a sector APB with centre P and a radius 10 cm.



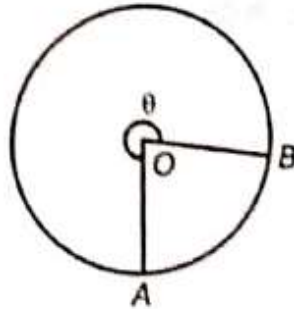
Cari / Find

- (a) jejari semi bulatan itu
the radius of the semicircle
 (b) perimeter rantau berlorek itu.
the perimeter of the shaded region

[4 markah/marks]

[Jawapan (a) 6.442 cm (b) 34.23 cm]

5. Rajah menunjukkan satu bulatan dengan pusat O dan berjejari 7 cm. Diberi bahawa panjang lengkok minor AB ialah 10 cm, cari
The diagram shows a circle with centre O and radius 7 cm. Given that the length of the minor arc AB is 10 cm, find

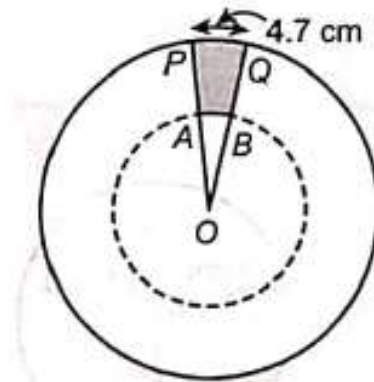


- (a) sudut θ , dalam radian
the angle θ , in radians
(b) luas sector minor
the area of the minor sector

[3 markah/marks]

[Jawapan : (a) 4.855 rad (b) 35 cm²]

6.



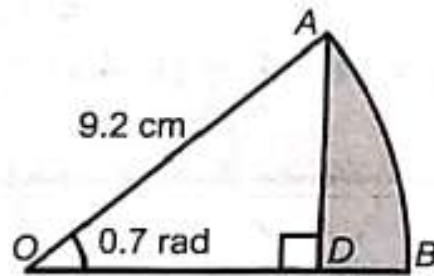
Rajah menunjukkan dua bulatan berpusat O dan masing-masing mempunyai jejari 8 cm dan 14 cm. Cari perimeter rantau berlerek itu.

The diagram shows two circles with centre O and with radii 8 cm and 14 cm respectively. Find the perimeter of the shaded region.

[3 markah/marks]

[Jawapan: 19.39 cm]

7.



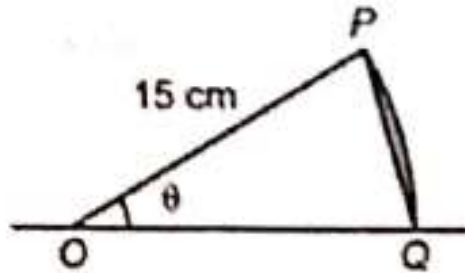
Rajah menunjukkan sebuah sektor AOB berpusat O dan mempunyai jejari 9.2 cm.
Cari perimeter rantau berlorek itu.

The diagram shows a sector AOB with centre O and radius 9.2 cm. Find the perimeter of the shaded region.

[3 markah/marks]

[Jawapan : 14.54 cm]

8. Rajah menunjukkan sebuah sektor berpusat O.
Diagram shows a sector with centre O.



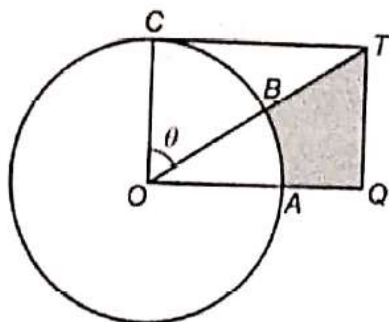
Diberi bahawa panjang perentas ialah 8 cm, cari
Given that the length of the chord is 8 cm, find

- (a) sudut θ , dalam radian
the angle θ , in radians
(b) luas rantau berlorek itu.
the area of the shaded region.

[3 markah/marks]

[Jawapan : (a) 0.5399 rad (b) 2.915 cm²]

9. Rajah menunjukkan sebuah bulatan berpusat O dengan jejari 5 cm. Diberi OQTC ialah sebuah segiempat tepat dengan luas 40 cm^2 , cari
The diagram shows a circle with centre O and radius 5 cm. Given OQTC is a rectangle with an area of 40 cm^2 , find

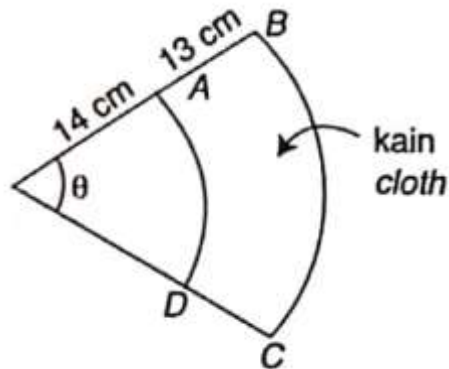


- (a) nilai θ , dalam radian
the value of θ , in radians
 (b) luas sektor AOB
the area of the sector AOB
 (c) perimeter kawasan berlorek
the perimeter of the shaded region

[4 markah/marks]

[Jawapan: (a) 1.012 rad (b) 7.01 cm^2 (c) 15.24 cm]

10. Rajah menunjukkan sebuah kipas tangan berbentuk sebuah sektor dan bahagian berlabel ABCD yang diperbuat daripada kain.
The diagram shows a fan in a form of a sector and the area labeled ABCD is made of cloth.



Diberi perimeter kain ialah $\left(\frac{41}{4\pi} + 26\right)$ cm, cari

Given the perimeter of the cloth is $\left(\frac{41}{4\pi} + 26\right)$ cm, find

- (a) sudut θ
the angle θ
- (b) luas kain itu dalam sebutan π
the area of the cloth used in terms of π

[4 markah/marks]

[Jawapan: (a) $\pi/4$ rad (b) $66\frac{5}{8}\pi$ cm²]

PEMBEZAAN

Idea had: $\lim_{x \rightarrow a} f(x) = L$

Pembezaan dengan prinsip pertama

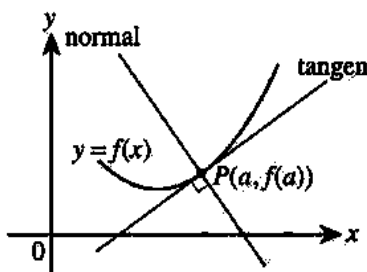
Jika $y = f(x)$, maka $\frac{dy}{dx} = \lim_{\delta x \rightarrow 0} \frac{\delta y}{\delta x}$, dengan δy ialah perubahan kecil dalam y dan δx ialah perubahan kecil dalam x .

Rumus pembezaan

- Jika $y = ax^n$, dengan a ialah pemalar dan n ialah integer, maka $\frac{d}{dx}(ax^n) = anx^{n-1}$.
- Jika y ialah fungsi bagi u dan u ialah fungsi bagi x , maka $\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$ (Petua rantai)
- Jika u dan v ialah fungsi bagi x , maka $\frac{d}{dx}(uv) = u \frac{dv}{dx} + v \frac{du}{dx}$ (Petua hasil darab)
- $\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2}$ (Petua hasil bahagi)

Aplikasi

Tangen dan normal

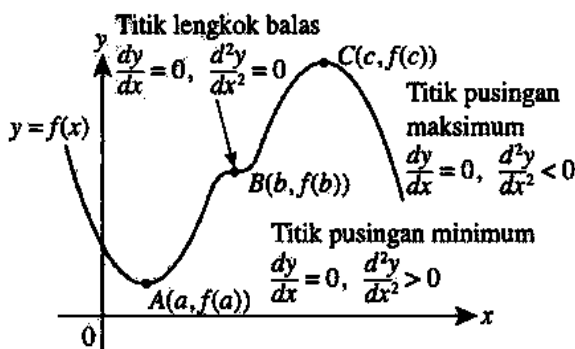


- Tangen: $y - f(a) = f'(a)(x - a)$
- Normal: $y - f(a) = -\frac{1}{f'(a)}(x - a)$

Kadar perubahan yang terhubung
Jika dua pemboleh ubah yang terhubung x dan y berubah dengan masa, t , maka

$$\frac{dy}{dt} = \frac{dy}{dx} \times \frac{dx}{dt}$$

Titik pegun bagi lengkung $y = f(x)$



Perubahan kecil dan penghampiran
Jika $y = f(x)$ dan perubahan kecil dalam x , iaitu δx menyebabkan perubahan kecil dalam y , iaitu δy , maka

$$\begin{aligned} \frac{\delta y}{\delta x} &\approx \frac{dy}{dx} \\ \delta y &\approx \frac{dy}{dx} \times \delta x \end{aligned}$$

$$\begin{aligned} \text{dan } f(x + \delta x) &\approx y + \delta y \\ &\approx y + \frac{dy}{dx}(\delta x) \end{aligned}$$

DIFFERENTIATION

The idea of limits: $\lim_{x \rightarrow a} f(x) = L$

Differentiation by first principles

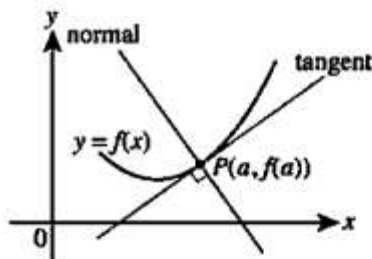
If $y = f(x)$, then $\frac{dy}{dx} = \lim_{\delta x \rightarrow 0} \frac{\delta y}{\delta x}$,
 where δy is a small change in y
 and δx is a small change in x .

Differentiation formula

- If $y = ax^n$, where a is a constant and n is an integer, then $\frac{d}{dx}(ax^n) = anx^{n-1}$.
- If y is a function of u and u is a function of x , then $\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$ (Chain rule)
- If u and v are functions of x , then
 - $\frac{d}{dx}(uv) = u \frac{dv}{dx} + v \frac{du}{dx}$ (Product rule)
 - $\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2}$ (Quotient rule)

Applications

Tangent and normal

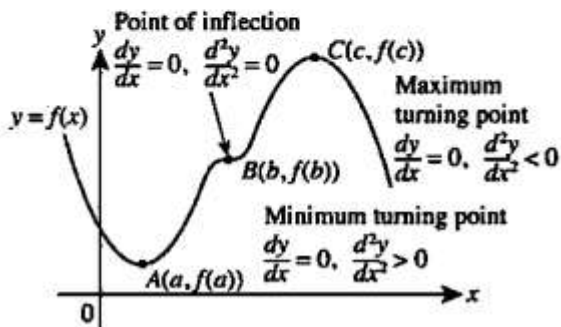


- Tangent: $y - f(a) = f'(a)(x - a)$
- Normal: $y - f(a) = -\frac{1}{f'(a)}(x - a)$

Rates of change of related quantities
 If two variables, x and y change with time, t , then

$$\frac{dy}{dt} = \frac{dy}{dx} \times \frac{dx}{dt}$$

Stationary points of curve $y = f(x)$



Small changes and approximations
 If $y = f(x)$ and the small change in x , that is δx , causes a small change in y , that is δy , then

$$\frac{\delta y}{\delta x} \approx \frac{dy}{dx}$$

$$\delta y \approx \frac{dy}{dx} \times \delta x$$

and $f(x + \delta x) \approx y + \delta y$
 $\approx y + \frac{dy}{dx}(\delta x)$

BAB 2 (TINGKATAN 5)
PEMBEZAAN / DIFFERENTIATION

1. Selesaikan setiap had yang berikut.
Evaluate the following limits.

a) $\lim_{x \rightarrow 0} \left(\frac{5x - x^2}{x} \right)$

(Jawapan : 5)

b) $\lim_{x \rightarrow 1} \left(\frac{x^2 - x}{x^2 + x - 2} \right)$

(Jawapan : $\frac{1}{3}$)

Jawapan :

a)

b)

2. Bezakan setiap fungsi yang berikut terhadap x .
Differentiate each of the following function with respect to x .

a) $f(x) = \frac{10}{x^2}$

(Jawapan : $-\frac{20}{x}$)

b) $f(x) = \frac{(x-1)(2x+5)}{x}$

(Jawapan : $2 + \frac{5}{x^2}$)

Jawapan :

a)

b)

3. Jika $f(x) = \frac{2x^2+10}{x-5}$, cari $f'(x)$.
If $f(x) = \frac{2x^2+10}{x-5}$, find $f'(x)$.

(Jawapan : $\frac{2x^2-20x-10}{(x-5)^2}$)

Jawapan :

4. Diberi bahawa $\frac{dy}{dx} = 7x^2 - \frac{3}{x} + 21$, cari $\frac{d^2y}{dx^2}$.
Given that $\frac{dy}{dx} = 7x^2 - \frac{3}{x} + 21$, find $\frac{d^2y}{dx^2}$.

(Jawapan : $14 - \frac{3}{x^3}$)

Jawapan :

5. Diberi bahawa $y = m^2 + 5$ dan $x = m - 10$. cari $\frac{dy}{dx}$ dalam sebutan x .
Given that $y = m^2 + 5$ and $x = m - 10$. Find $\frac{dy}{dx}$ in terms of x .

(Jawapan : $2x + 20$)

Jawapan :

6. Diberi bahawa persamaan garis lengkung ialah $y = 2x^3 + 5x^2 - 11x + 5$. Cari kecerunan tangen bagi garis tersebut pada titik (4, 6).
Given that the equation of the curve is $y = 2x^3 + 5x^2 - 11x + 5$. Find the gradient of the tangent at (4, 6).

(Jawapan : 125)

Jawapan :

7. Diberi titik $Q(1, -2)$ terletak di atas garis lengkung $y = 5x^2 - 19x - 30$.
Given that a point $Q(1, -2)$ lies on the curve $y = 5x^2 - 19x - 30$.

- a) Cari kecerunan tangen garis lengkung tersebut pada titik Q .
Find the gradient of the tangent of the curve at point Q .

(Jawapan : - 9)

- b) Cari persamaan normal pada titik Q .
Find the equation of the normal at point Q .

(Jawapan : $y = \frac{x}{9} - \frac{19}{9}$)

Jawapan :

a)

b)

8. Cari persamaan normal kepada lengkung $y = \frac{48}{\sqrt{x}}$ pada titik A (4, 0).
Find the equation of the normal to the curve $y = \frac{48}{\sqrt{x}}$ at point A (4, 0).

(Jawapan : $y = \frac{x}{3} - \frac{4}{3}$)

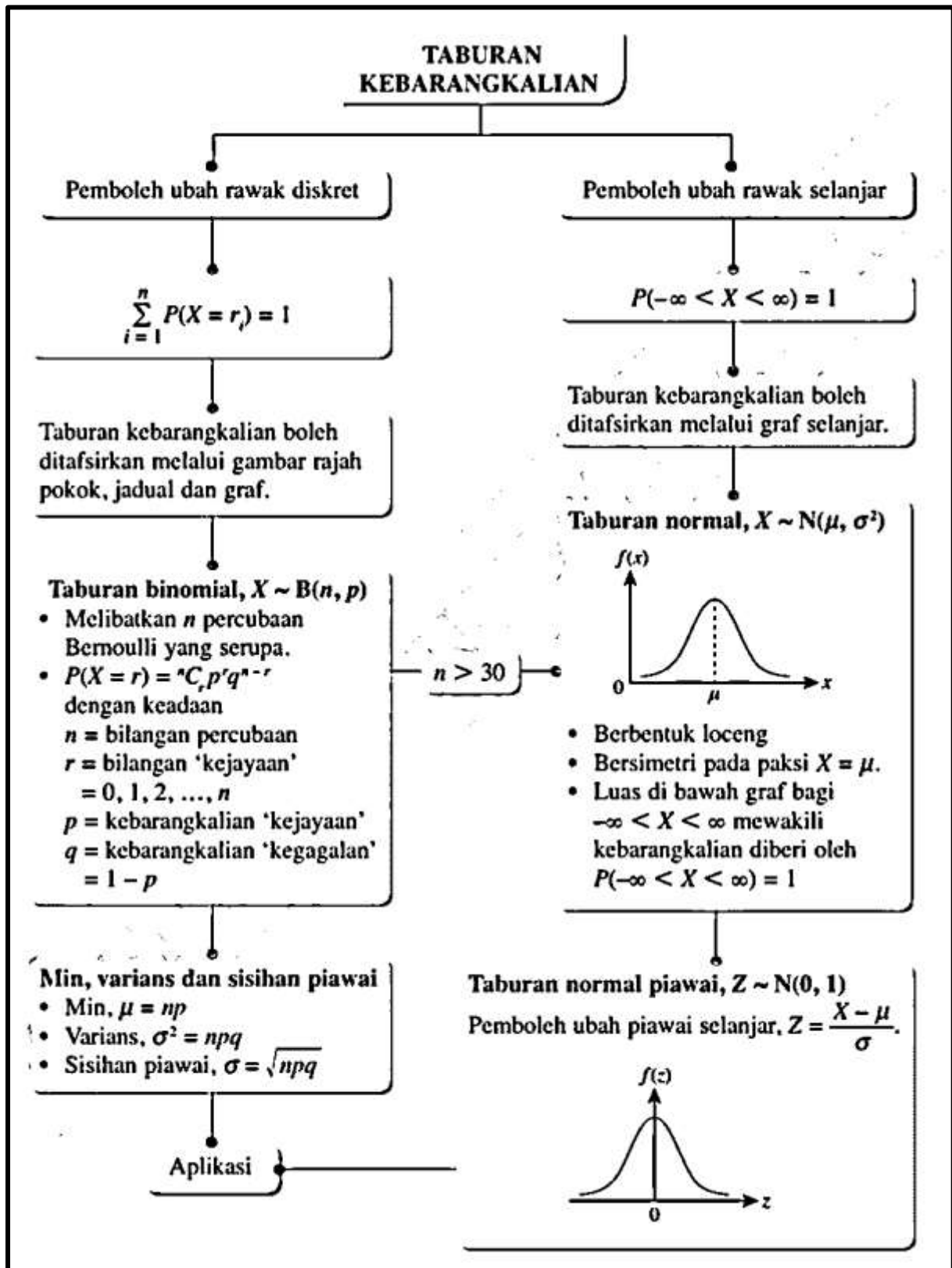
Jawapan :

9. Aminah melukis satu corak berbentuk lengkung pada sehelai kertas. Diberi persamaan garis lengkung bagi corak tersebut ialah $y = \frac{1}{3}x^3 - x^2 - 9$. Beliau ingin melukis corak bunga pada titik pusingan lengkung tersebut. Tentukan koordinat corak bunga yang akan dilukis oleh Aminah.

Aminah draws a curved pattern on a piece of paper. Given that the equation of the curve for the pattern is $y = \frac{1}{3}x^3 - x^2 - 9$. He wanted to draw a floral pattern on the turning point of the curve. Determine the coordinates of the flower pattern that Aminah will draw.

(Jawapan : (0, - 9) & (2, $-\frac{31}{3}$))

Jawapan :



PROBABILITY DISTRIBUTION

Discrete random variable

$$\sum_{i=1}^n P(X = r_i) = 1$$

The probability distribution can be interpreted by a tree diagram, a table or a graph.

Binomial distribution, $X \sim B(n, p)$

- Involves n Bernoulli trials which are similar.
- $P(X = r) = {}^n C_r p^r q^{n-r}$ where
 n = number of trials
 r = number of 'success'
 $= 0, 1, 2, \dots, n$
 p = probability of 'success'
 q = probability of 'failure'
 $= 1 - p$

$n > 30$

Mean, variance and standard deviation

- Mean, $\mu = np$
- Variance, $\sigma^2 = npq$
- Standard deviation, $\sigma = \sqrt{npq}$

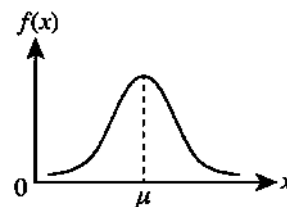
Applications

Continuous random variable

$$P(-\infty < X < \infty) = 1$$

The probability distribution can be interpreted by using a continuous graph.

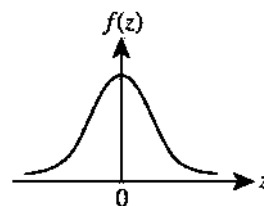
Normal distribution, $X \sim N(\mu, \sigma^2)$



- Bell-shaped
- Symmetrical at $X = \mu$ axis.
- Area under the graph for $-\infty < X < \infty$ represents the probability which is given by $P(-\infty < X < \infty) = 1$

Standard normal distribution, $Z \sim N(0, 1)$

A standard continuous random variable,
 $Z = \frac{X - \mu}{\sigma}$.



BAB 5 (TINGKATAN 5)

TABURAN KEBARANGKALIAN / PROBABILITY DISTRIBUTIONS

1. Dalam suatu peperiksaan , didapati bahawa 70% daripada calon lulus dalam peperiksaan tersebut. Jika 10 orang calon dipilih secara rawak, cari kebarangkalian bahawa

In an examination, it is found that 70% of the candidates passed the examination. If 10 candidates are chosen at random, find the probability that

- a) tepat 8 orang calon lulus dalam peperiksaan tersebut,
exactly 8 candidates passed the examination,

(2 markah / 2 marks)

- b) sekurang-kurangnya 2 orang calon lulus dalam peperiksaan tersebut
at least 2 candidates passed the examination

(2 markah/ 2 marks)

(Jawapan : (a) 0.2335 (b) 0.9999)

Jawapan / Answer

a)

b)

2. Pemboleh ubah rawak X mempunyai taburan normal min 25 dan sisihan piawai σ .
Diberi skor-z ialah 3.5 apabila $X = 26.75$
The random variable X has a normal distribution with a mean of 25 and a standard deviation of σ . It is given the z- score is 3.5 when $X = 26.75$

Cari/ find

- a) nilai bagi σ
the value of σ

(2 markah/ 2 marks)

- b) nilai bagi k dengan keadaan $P(X > k) = 0.7172$
value of k if $P(X > k) = 0.7172$

(3 markah/ 3 marks)

(Jawapan : (a) 0.5 (b) 24.713)

Jawapan / Answer

- a)

- b)

3. Firdaus membeli 10 biji mangga. Kebarangkalian sebiji mangga yang dibelinya manis ialah 0.2. Kebarangkalian bahawa mangga itu:

Firdaus bought 10 mangoes. The probability that an mango bought is sweet 0.2. Find probability that these mangoes

- a) tepat sebiji manis ialah
exactly one mango is sweet

(2 markah/2 marks)

- b) sekurang-kurangnya dua biji manis
at least 2 mangoes are sweet

(2 markah/2 marks)

(Jawapan : (a) 0.2684 (b) 0.6242)

Jawapan / Answer

a)

b)

4. Dalam suatu kaji selidik daripada 100 orang murid, 40 orang memakai cermin mata. Jika 10 orang murid dipilih secara rawak, apakah kebarangkalian kurang daripada 2 memakai cermin mata?

In a survey, out of 100 students , 40 of them wearing spectacles. If 10 students are chosen at random, what is the probability that less than 2 students wearing spectacles?

(3 markah/ 3 marks)

(Jawapan : 0.04636)

Jawapan / Answer

5. Pemboleh ubah rawak X mengikut suatu taburan binomial dengan 10 cubaan dengan keadaan kebarangkalian kejayaan dalam setiap cubaan ialah p . Min bilangan kejayaan ialah 4. Hitung

A random variable X has a binomial distribution with 10 trials where the probability of success in each trials is p . The mean number of successes is 4. Calculate

- a) nilai p / *the value of p*

(2 markah / 2 marks)

- b) $P(X \leq 2)$

(3 markah/ 3 marks)

(Jawapan : (a) $\frac{2}{5}$ (b) 0.1673)

Jawapan / Answers

- a)

- b)

6. Pemboleh ubah rawak X mewakili taburan binomial dengan 10 percubaan dan kebarangkalian berjaya ialah $\frac{1}{3}$. Cari:

The random variable X represents a binomial distribution with 10 trials and the probability of success is $\frac{1}{3}$. Find

- a) sisihan piawai taburan itu. / *the standard variation of the distribution*

(2 markah / 2 marks)

- b) kebarangkalian bahawa sekurang-kurangnya satu percubaan adalah berjaya.
the probability that at least one trial is success

(2 markah/ 2 marks)

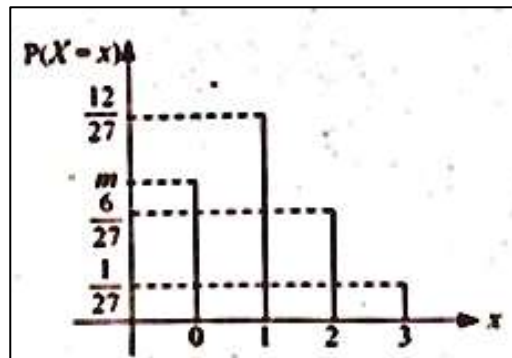
(Jawapan : (a) 1.491 (b) 0.9827)

Jawapan / Answer

- a)

- b)

7. Rajah di bawah menunjukkan graf suatu taburan binomial bagi X. Cari
Diagram below shows the graph of binomial distribution for X. Find



a) $P(X \geq 1)$

(1 markah / 1 mark)

b) nilai m
the value of m

(2 markah/ 2 marks)

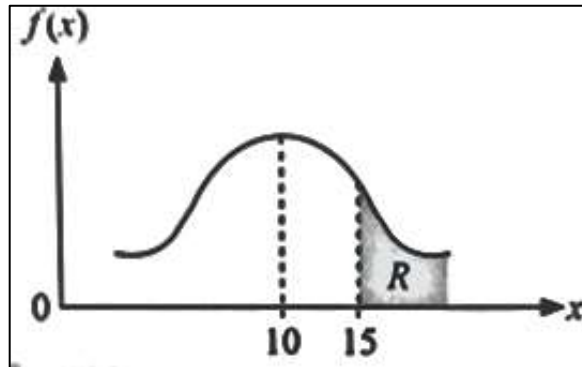
(Jawapan : (a) $\frac{19}{27}$ (b) $\frac{8}{27}$)

Jawapan / Answer

a)

b)

8. Rajah menunjukkan graf taburan normal bagi pemboleh ubah rawak selanjar X .
Diagram shows a normal distribution graph for a continuous random variable X .



- a) Nyatakan min bagi X
State the mean of X (1 markah/ 1 mark)
- b) Ungkapkan rantau berlorek R dalam tatatanda kebarangkalian.
Express the shaded region R in probability notations. (1 markah / 1 mark)
- c) Jika $P(X < 15) = 0.6352$, cari $P(X > 15)$
If $P(X < 15) = 0.6352$, find $P(X > 15)$ (2 markah/2 marks)
- (Jawapan : (b) $P(X > 15)$ (c) 0.3648)

Jawapan /Answer

- a)
- b)
- c)

9. Didapati bahawa 80% daripada graduan universiti di sebuah negeri mendapat pekerjaan. Jika 10 orang graduan universiti dari negeri itu dipilih secara rawak, cari kebarangkalian bahawa

It is found that 80% of university graduates in a state are employed. If 10 university graduates from the state are selected at random, find the probability that

- a) tepat 9 orang mendapat pekerjaan
exactly 9 of them are employed

(2 markah/2 marks)

- b) selebih-lebihnya 2 orang tidak mendapat pekerjaan
at most 2 of them are unemployed.

(3 markah/ 3 marks)

(Jawapan : (a) 0.2684 (b) 0.6778)

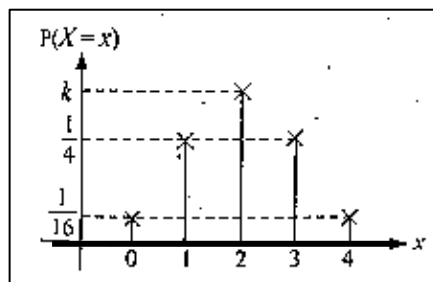
Jawapan / Answer

a)

b)

10. Pemboleh ubah rawak diskret X mempunyai satu taburan kebarangkalian binomial dengan $n = 4$ dengan keadaan n ialah bilangan percubaan. Rajah di bawah menunjukkan taburan kebarangkalian bagi X . Cari

The discrete random variable X has a binomial probability distribution with $n=4$, where n is the number of trials. Diagram below shows the probability distribution of X .



- a) Nilai k / *the value of k*

(2markah/ 2 marks)

- b) $P(X \geq 3)$

(3 markah/ 2 marks)

(Jawapan : (a) $\frac{3}{8}$ (b) $\frac{5}{16}$)

Jawapan / Answer

a)

b)

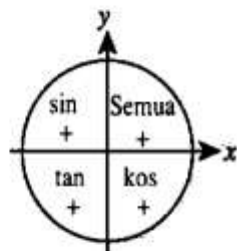
FUNGSI TRIGONOMETRI

Mewakilkkan sudut positif dan sudut negatif dalam satah Cartes.

- Unit darjah dan radian.
- Sudut pada bulatan penuh ialah 360° .

Menentukan nisbah trigonometri bagi sebarang sudut:

- Enam fungsi trigonometri
- Sudut rujukan
- Tanda nisbah trigonometri dalam 4 sukuan



- Melukis dan melakar graf fungsi trigonometri.
- Kesan perubahan a , b dan c pada graf berikut:
 $y = a \sin bx + c$
 $y = a \cos bx + c$
 $y = a \tan bx + c$
- Mencari penyelesaian dan menentukan bilangan penyelesaian.

Identiti trigonometri

- Rumus sudut pelengkap
- Identiti asas
- Rumus sudut majmuk
- Rumus sudut berganda
- Rumus sudut separuh

Aplikasi

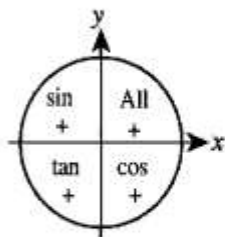
TRIGONOMETRIC FUNCTIONS

Represent positive angles and negative angles in a Cartesian plane

- Angles in degrees or radians.
- Angle in a full circle is 360° .

Determine the trigonometric ratios of any angle:

- Six trigonometric functions
- Reference angle
- Signs for the trigonometric ratios in the 4 quadrants



- Draw and sketch graphs of trigonometric functions.
- Effects of changing a , b and c on the following graphs:
 $y = a \sin bx + c$
 $y = a \cos bx + c$
 $y = a \tan bx + c$
- Find the solutions and determine the number of solutions.

Trigonometric identities

- Complementary angle formulae
- Basic identities
- Addition formulae
- Double angle formulae
- Half angle formulae

Applications

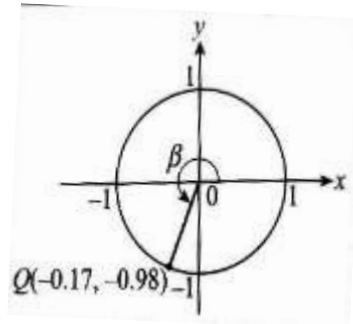
BAB 6 (TINGKATAN 5)

FUNGSI TRIGONOMETRI / TRIGONOMETRIC FUNCTIONS

1. Cari nilai sudut β bagi bulatan unit berikut.

Find the values of angle of β for the unit circle given.

- (a) $\sin \beta$
- (b) $\cos \beta$
- (c) $\tan \beta$



Jawapan : (a) -0.98 ; (b) -0.17 ;(c) 5.7647

2. Tanpa menggunakan kalkulator, tentukan nilai berikut:

Without using calculator, determine the following values:

- (a) $\cos^2 \frac{\pi}{4} - \sin \frac{\pi}{2}$
- (b) $\frac{\sin 30^\circ \cos 60^\circ}{\cos 45^\circ - \tan 45^\circ}$

Jawapan : (a) $-\frac{1}{2}$ (b) $\frac{1}{2(\sqrt{2}-2)}$

3. Cari nilai a , b dan c bagi fungsi $y = a \sin bx + c$, jika
Find the values of a , b and c for the function $y = a \sin bx + c$, if
Amplitude = 3, tempoh = 2π dan paksi utama, $y = 2$.
Amplitude = 3, period = 2π and the main axis, $y = 2$.

Jawapan : $a = 3, b = 1, c = 2$

4. Kembang dan ringkaskan.
Expand and simplify.

$$(\cos \theta - \sin \theta)^2$$

Jawapan : $1 - \sin 2\theta$

5. Selesaikan yang berikut bagi $0 \leq x \leq 360^\circ$

Solve the following for $0 \leq x \leq 360^\circ$

$$\cos x + \sin 2x = 0$$

Jawapan : $x = 90^\circ, 210^\circ, 270^\circ, 330^\circ$

6. Diberi $\tan x = \frac{5}{12}$ dan $90^\circ < x < 360^\circ$, cari nilai bagi $\cos x$.

Given $\tan x = \frac{5}{12}$ and $90^\circ < x < 360^\circ$, find the value of $\cos x$.

Jawapan : $-\frac{12}{13}$

7. Diberi $\sin 30^\circ = 0.5$, $\cos 30^\circ = 0.8660$ dan $\tan 30^\circ = 0.5774$. Cari nilai-nilai fungsi trigo berikut tanpa menggunakan kalkulator.

Given that $\sin 30^\circ = 0.5$, $\cos 30^\circ = 0.8660$ and $\tan 30^\circ = 0.5774$. Find the values of the trigonometric function without using calculator.

(a) $\operatorname{cosec} 30^\circ$ / $\operatorname{cosec} 30^\circ$

(b) $\sin 60^\circ$

Jawapan : (a) 2; (b) 0.8660

8. Diberi $\sin A = -\frac{15}{17}$ dan $\tan B = -\frac{4}{3}$ dengan $180^\circ \leq A \leq 270^\circ$ dan $270^\circ \leq B \leq 360^\circ$, cari

Given $\sin A = -\frac{15}{17}$ and $\tan B = -\frac{4}{3}$ where $180^\circ \leq A \leq 270^\circ$ and $270^\circ \leq B \leq 360^\circ$, find

(a) $\cos(B - A)$

$\cos(B - A)$

(b) $\cot(A + B)$

$\cot(A + B)$

Jawapan : (a) $\frac{36}{85}$ (b) $\frac{84}{13}$

9. Ungkapkan setiap sudut berikut dalam sebutan sudut rujukannya.

Express the angles in terms of its reference angle.

(a) $\sin 230^\circ$

(b) $\cos(-140^\circ)$

Jawapan : (a) $-\sin 50^\circ$; (b) $-\cos 40^\circ$

10. Selesaikan persamaan $2 \sin 2\theta = 1$, $0 \leq \theta \leq 360^\circ$

Solve the equation of $2 \sin 2\theta = 1$, $0 \leq \theta \leq 360^\circ$

Jawapan : $15^\circ, 75^\circ, 195^\circ, 255^\circ$